LOGIC

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SECOND EDITION

PREFACE

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THIS book is a reissue with alterations and additions of my *Deductive Logic* published towards the close of 1888. It is now called by a wider name because the treatment of Inductive Inference has been included.

My main obligations in preparing the new edition are to Mr. J. N. Keynes, who seems justly entitled to become one of our chief legislators on logical terminology.

In a work which appeared at the same time as my own Father Clarke laid the English-speaking world under his debt by an exposition of the *Scholastic Logic* in which there is not a dull page from beginning to end.

Other books which I should like to mention are Shute's *Discourse on Truth*, Welton's *Manual of Logic*, Minto's *Logic, Inductive and Deductive*, and the new edition of the *Palaestra Logica*.

I have to thank Mr. E. L. Hawkins of Merton College, and Mr. F. G. Brabant of Corpus Christi College, Oxford, for some helpful criticisms while the work was in progress. The keen eyes of Mr. George Holden, sub-librarian of All Souls' College, detected some mistakes in the first edition which had escaped other observation.

16 MUSEUM ROAD, OXFORD,
Jan. 9, 1900.

PREFACE TO DEDUCTIVE LOGIC

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One critic, who was kind enough to look at this book in manuscript, recommended me to abandon the design of publishing it, on the ground that my logic was too like all other logics; another suggested to me to cut out a considerable amount of new matter. The latter advice I have followed; the former has encouraged me to hope that I shall not be considered guilty of wanton innovation. The few novelties which I have ventured to retain will, I trust, be regarded as legitimate extensions of received lines of teaching.

My object has been to produce a work which should be as thoroughly representative of the present state of the logic of the Oxford Schools as any of the text-books of the past. The qualities which I have aimed at before all others have been clearness and consistency. For the task which I have taken upon myself I may claim one qualification—that of experience; since more than seventeen years have now elapsed since I took my first pupil in logic for the Honour School of Moderations, and during that time I have been pretty continuously engaged in studying and teaching the subject.

In acknowledging my obligations to previous writers I must begin with Archbishop Whately, whose writings first gave me an interest in the subject. The works of

Mill and Hamilton have of course been freely drawn upon. I have not followed either of those two great writers exclusively, but have endeavoured to assimilate what seemed best in both. To Professor Fowler I am under a special debt. I had not the privilege of personal teaching from him in logic, as I had in some other subjects; but his book fell into my hands at an early period in my mental training, and was so thoroughly studied as to have become a permanent part of the furniture of my mind. Much the same may be said of my relation to the late Professor Jevons's *Elementary Lessons in Logic.* Two other books, which I feel bound to mention with special emphasis, are Mansel's edition of Aldrich and McCosh's *Laws of Discursive Thought.* If there be added to the foregoing Watts's *Logic,* Thomson's *Outlines of the Laws of Thought,* Bain's *Deductive Logic,* Jevons's *Studies in Deductive Logic and Principles of Science,* Bradley's *Principles of Logic,* Abbott's *Elements of Logic,* Walker's edition of Murray, Ray's *Text-book of Deductive Logic,* and Weatherley's *Rudiments of Logic,* I think the list will be exhausted of modern works from which I am conscious of having borrowed. But, not to forget the sun, while thanking the manufacturers of lamps and candles, I should add that I have studied the works of Aristotle according to the measure of my time and ability.

This work has had the great advantage of having been revised, while still in manuscript, by Mr. Alfred Robinson, Fellow of New College, to whom I cannot sufficiently

express my obligation. I have availed myself to the full of the series of criticisms which he was kind enough to send me. As some additions have been made since then, he cannot be held in any way responsible for the faults which less kindly critics may detect.

For the examples at the end I am mainly indebted to others, and to a large extent to my ingenious friend, the Rev. W. J. Priest of Merton College.

My thanks are due also to my friend and former pupil, Mr. Gilbert Grindle, Scholar of Corpus, who has been at the pains to compose an index, and to revise the proofs as they passed through the press.

And last, but not least, I must set on record my gratitude to Commander R. A. Stock, R.N., one of Her Majesty's Knights of Windsor, without whose brotherly aid this work might never have been written, and would certainly not have assumed exactly its present shape.

OXFORD,
    October 22, 1888.

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INTRODUCTION.

§ 1. Logic is the Science of the Laws of Thought.

§ 2. Thought, as here used, is confined to the faculty of comparison. All thought involves comparison, that is to say, a recognition of likeness or unlikeness.

§ 3. The laws of thought are the conditions of correct thinking. The word 'law' however is so ambiguous that it will be well to determine more precisely in what sense it is here used.

§ 4. We talk of the 'laws of the land' and of the 'laws of nature,' and it is evident that we mean very different things by these expressions. By a law in the political sense is meant a command imposed by a superior upon an inferior, and sanctioned by a penalty for disobedience. But by the 'laws of nature' are meant merely certain uniformities among natural phenomena; for instance, the 'law of gravitation' means that every particle of matter does invariably attract every other particle of matter in the universe with a force in direct proportion to the mass and in inverse proportion to the square of the distance.

The word 'law' is transferred by a metaphor from one of these senses to the other. The effect of such a command as that described above is to produce a certain

amount of uniformity in the conduct of men, and so, where we observe uniformity in nature, we assume that it is the result of such a command, whereas the only thing really known to us is the fact of uniformity itself. Now in which of these two senses are we using the expression 'laws of thought?' The laws of the land, it is plain, are often violated, whereas the laws of nature never can be so [1]. Can the laws of thought be violated in like manner with the laws of the land? Or are they inviolable like the laws of nature?

In appearance they can be, and manifestly often are violated. For how else could error be possible? But in reality they can not. No man ever accepts a contradiction when it presents itself to the mind as such: but owing to confusion of thought and the infinite perplexities of language conclusions are often reached which contain a latent contradiction. It is the business of logic to bring these conclusions to the test of first principles. In this way logic acts as the touchstone of truth.

The laws of thought then in their ultimate expression are certain uniformities which invariably hold among mental phenomena, and so far they resemble the laws of nature: but in appearance they may be violated owing to error, as the laws of the land may be violated by crime.

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[1] There is a sense in which people frequently speak of the laws of nature being violated, as when one says that intemperance or celibacy is a violation of the laws of nature, but here by 'nature' is meant an ideal perfection in the conditions of existence.

§ 5. Logic is often defined as the science of the *formal* laws of thought. We have avoided that expression because it suggests that there are *material* laws of thought, a phrase to which it would be difficult to assign a meaning. It is not equivalent to 'laws relating to the matter of thought.' With such laws logic is not concerned, unless they are so universal as to apply to every object thought about, of whatever nature it may be, in which case they are considered to belong to the form and not the matter.

The distinction between form and matter is one which pervades all nature. We are familiar with it in the case of manufactured articles. A cup, for instance, with precisely the same form, may be composed of very different matter—gold, silver, pewter, horn, or what not.

Similarly in every act of thought we may distinguish two things—

(1) the object thought about,

(2) the way in which the mind thinks of it.

The first is called the Matter; the second the Form of Thought. Thus I may judge 'Money-lenders are rogues,' or 'Porpoises are playful.' The matter is in these two cases quite different, but the form is the same, namely, a judgement.

Since the form may be the same whilst the matter is different, we may say that logic is concerned only with the way in which the mind thinks, and has nothing to do with the particular objects thought about. In other words, logic is concerned with the essential and necessary elements of thought as opposed to such as are accidental

and contingent. By 'contingent' is meant what holds true in some cases, but not in others. For instance, in the particular case of equilateral triangles it is true to say, not only that 'All equilateral triangles are equiangular,' but also that 'All equiangular triangles are equilateral.' But the evidence for these two propositions is distinct. The one is not a formal consequence of the other. If it were, we should be able to apply the same inference to all matter and assert universally that if All S is P, All P is S, which it is notorious that we cannot do.

§ 6. We have defined logic as a *science.* Is it correct to call it also an *art*? The answer to this question must depend upon the meaning we assign to those terms.

Broadly speaking, there is the same difference between Science and Art as there is between knowing and doing.

> Science is systematized knowledge;
> Art is systematized action.
>
> Science is acquired by study;
> Art is acquired by practice.

Now logic is manifestly a branch of knowledge, and does not necessarily confer any practical skill. It is only the right use of its rules in thinking which can make men think better. It is therefore in the broad sense of the terms wholly a science and not at all an art.

But the word 'art,' like most others, is ambiguous, being often used not for skill displayed in practice, but for the knowledge necessary thereto. This meaning is better conveyed by the term 'practical science.'

Science is either Speculative or Practical. In the first case we study merely that we may know; in the latter that we may do.

> Anatomy is a speculative science;
> Surgery is a practical science.

In the first case we study the human frame in order that we may understand its structure; in the second that we may assist its needs. Whether logic be a speculative or a practical science depends upon the way in which it is treated. If we study the laws of thought merely that we may know what they are, we are making it a speculative science; if we study the same laws with a view to deducing rules for the guidance of thought, we are making it a practical science.

Hence logic may be declared to be both the science and the art of thinking. It is the art of thinking in the same sense in which grammar is the art of speaking. Grammar is not in itself the right use of words, but a knowledge of it enables men to use words correctly. In the same way a knowledge of logic enables men to think correctly, or at all events to avoid incorrect thoughts.

§ 7. *The Three Fundamental Laws of Thought.*

The laws of thought are all reducible to the three following axioms, which are known as—

(1) The Law of Identity

> Whatever is is.
> Every A is A.

(2) The Law of Contradiction

> Nothing can both be and not be.
> No A is not A.

(3) The Law of Excluded Middle

> Everything must either be or not be
> Everything is either A or not A.

At first sight these three principles seem to be deriv-
able one from the other. For is not the second, it may
be asked, the obverse and the third the converse by
contraposition of the first[1]? And are not obversion and
contraposition valid forms of inference? Yes, we shall
afterwards have to recognise them as such. But the
validity of these modes of inference depends upon the
laws of thought. We cannot therefore make the laws of
thought depend upon them.

In reality each of these principles is independent and
self-evident.

If it were possible for the law of identity to be
violated, no violation of the law of contradiction would
necessarily ensue : for a thing might then be something
else, without being itself at the same time, whereas
what the law of contradiction forbids is that a thing
should be *both* itself and something else. Neither would
the law of excluded middle be infringed. For, on the
supposition, a thing would be something else. Now

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[1] (1) All A is A.

   (2) No A is not-A (obverse of 1).

   (3) All but A is not-A (converse by contraposition of 1).

all that the law of excluded middle demands is that it should either be itself or not. A would in this case adopt the alternative of being not-A.

Again, the violation of the law of contradiction does not involve any violation of the law of identity; for a thing might in that case be still itself, even though, owing to the law of contradiction not holding, it were not itself at the same time. Neither would the law of excluded middle be infringed. For a thing would, on the supposition, be both itself and not itself, which is the very reverse of being neither.

Lastly, the law of excluded middle might be violated without a violation of the law of contradiction: for we should then have a thing which was neither A nor not-A, but not a thing which was both at the same time. Neither would the law of identity be infringed. For we should in this case have a thing which neither was nor was not, so that the conditions of the law of identity could not exist to be broken. That law postulates that whatever is, is: here we have a thing which never was to begin with.

These principles are of so simple a character that the discussion of them is apt to be regarded as puerile. Especially is this the case with regard to the law of identity. This principle in fact is one of those things which are 'more honoured in the breach than in the observance.' Suppose for a moment that this law did not hold—then what would become of all our reasoning? Where would be the use of establishing conclusions about

things, if they were able to evade us by a Protean change of identity?

The remaining two laws supplement each other in the following way. The law of contradiction enables us to affirm of two exhaustive and mutually exclusive alternatives that it is impossible for both to be true; the law of excluded middle entitles us to add that it is equally impossible for both to be false. Or, to put the same thing in a different form, the law of contradiction lays down that one of two such alternatives must be false; the law of excluded middle adds that one must be true.

§ 8. There are three acts or processes of thought with which logic is concerned—

    (1) Conceiving,
    (2) Judging,
    (3) Inferring.

To conceive is to grasp in the mind the idea or notion of anything. Hence Conception is also known as Simple Apprehension. I conceive a triangle when I grasp in my mind the idea of a figure contained by three sides. Conception, be it remembered, is an act of pure thought, and has nothing to do with sensuous imagination. We can conceive things of which we can form no mental picture. Conception is generally *accompanied* by imagination; but that is a different matter.

To judge is to pronounce as to the agreement or disagreement of two ideas. E.g. we have in our minds the idea of a cup and the idea of a thing made of porcelain, and we combine them in the judgement—

'This cup is made of porcelain;' or separate them in the judgement—'This cup is not made of porcelain.'

Judgement, it will be seen, is here confined to the comparison of ideas already formed in the mind. Sometimes the term is extended to the comparison of nameless sense-impressions. But this amounts to identifying judgement with thought in general.

To infer is to pass from one or more judgements to another. Thus from the several judgements, 'This, that, and the other thing made of porcelain is brittle,' we rise to the universal judgement, 'Whatever is made of porcelain is brittle;' or, having already formed that judgement, we add to it another, 'This cup is made of porcelain,' and so descend to the new judgement, 'This cup is brittle.'

§ 9. Corresponding to these three processes there are three products of thought, namely—

(1) The Concept,

(2) The Judgement,

(3) The Inference.

Since our language has a tendency to confuse the distinction between processes and products, it is the more necessary to keep them distinct in thought. We speak quite indiscriminately of Conception, Judgement, Inference, Sensation, Imagination, Sight, Thought, Division, Definition, and so on, whether we mean in any case a process or a product. The distinction is much better observed both in Latin and Greek than in English. Thus in Latin *visus* or *visio* is used for the act of seeing, and

visum for the thing seen; in Greek νόησις stands for the process, νόημα for the product of thought; αἴσθησις for the act of sensation, and αἰσθητόν for the object of sense, and so on in other cases.

§ 10. The direct object of logic is the study of the products rather than of the processes of thought. But at the same time in studying the products we are studying the processes in the only way in which it is possible to do so. For the human mind cannot be both actor and spectator at once; we must wait until a thought is formed in our minds before we can examine it. Thought must be already dead in order to be dissected; there is no vivisection of consciousness. Thus we can never know more of the processes of thought than what is revealed to us in their products.

§ 11. Primarily and necessarily logic is concerned with thought; secondarily and accidentally with language as the garment of thought. If thought remains at home and in bed, as it were, in the mind of an individual, it may perhaps dispense with language; but if it wishes to go out into society, it must put on its clothes, and array itself in speech or some other form of language. So much does the tailor make the man in this instance that the Greeks as a nation never distinguished between thought and language. Their word λόγος, from which 'logic' is derived, stands equally for both; it corresponds both to *ratio* and to *oratio* in Latin. The philosophers of course were obliged to distinguish between thought and its expression, which they did by calling one the

λόγος ἐνδιάθετος and the other the λόγος προφορικός, the one ὁ ἔσω or ὁ ἐν τῇ ψυχῇ λόγος, and the other ὁ ἔξω λόγος. This ambiguity in the name lends ground to a difference of opinion that has subsisted among logicians as to whether the subject-matter of their science is thought pure and simple or thought as expressed in language. Granting that it is thought pure and simple, it is obviously impossible to discuss it except as clothed in language.

§ 12. When the three products of thought are expressed in language, they are called respectively—

(1) The Term,

(2) The Proposition,

(3) The Inference [1].

§ 13. It has been declared that thought in general is the faculty of comparison, and we have seen that there are three products of thought. It follows that each of these products of thought must be the result of a comparison of some kind or other.

The term is the result of comparing attributes.

The proposition is the result of comparing terms.

The inference is the result of comparing propositions.

Not only are common terms the results of comparison, but singular terms, or the names of individuals, are so too.

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[1] Such is the ambiguity of language that we have already used the term 'inference' in three different senses—(1) for the act or process of inferring, (2) for the result of that act as it exists in the mind, (3) for the same thing as expressed in language. Later on we shall have sometimes to apply it to an inferred proposition, or what is otherwise known as the conclusion of an inference.

The earliest result of thought is the recognition of an individual object as such, that is to say, as distinguished and marked off from the mass of its surroundings. No doubt the first impression produced upon the nascent intelligence of an infant is that of a confused whole. It requires much exercise of thought to distinguish this whole into its parts. The completeness of the recognition of an individual object is announced by attaching a name to it. Hence even an individual name, or singular term, implies thought or comparison. Before the child can attach a meaning to the word 'mother,' which to it is a singular term, it must have distinguished between the set of impressions produced in it by one object from those which are produced in it by others. Thus, when Vergil says

> Incipe, parve puer, risu cognoscere matrem,

he is exhorting the beatific infant to the exercise of the faculty of comparison.

That a common term implies comparison does not need to be insisted on. It is because things resemble each other in certain of their attributes that we call them by a common name, and this resemblance could not be ascertained except by comparison at some time and by some one. Thus a common term is the compressed result of an indefinite number of comparisons, which lie wrapped up in it, like so many fossils, witnessing to prior ages of thought.

In the next product of thought, namely, the proposition, we have the result of a single act of comparison between

two terms. For this reason among others the proposition has been called the unit of thought, as being the simplest and most direct result of comparison. Just as in social science the unit is not the individual on the one hand nor the world at large on the other, but the state, which is intermediate between the two, so in logic the unit is neither the term nor the inference, but the proposition, which is intermediate between the two.

In the third product of thought, namely, the inference, we have a comparison of propositions. This need not be enlarged on at this point. For the present let us return to the first product of thought.

§ 14. The nature of singular terms has not given rise to much dispute, but the nature of common terms has been the great battle-ground of logicians. What corresponds to a singular term, at least when it is concrete, is easy to determine, for the thing of which it is a name is there to point to: but the meaning of a common term, like 'man' or 'horse' is not so obvious as people are apt to think on first hearing of the question.

A common term or class-name was known to mediæval logicians under the title of a Universal; and it was on the question 'What is a Universal?' that they split into the three schools of Realists, Nominalists, and Conceptualists. Here are the answers of the three schools to this question in their most exaggerated form—

Universals, said the Realists, are substances having an independent existence in nature.

Universals, said the Nominalists, are a mere matter of

words, the members of what we call a class having nothing in common but the name.

Universals, said the Conceptualists, exist in the mind alone. They are the conceptions under which the mind regards external objects.

The origin of pure Realism is due to Plato and his doctrine of ' ideas'; for Idealism, in this sense, is not opposed to Realism, but identical with it. Plato seems to have imagined that, as there was a really existing thing corresponding to a singular term, such as Socrates, so there must be a really existing thing corresponding to the common term ' man.' But when once the existence of these general objects is admitted, they swamp all other existences. For individual men are fleeting and transitory —subject to growth, decay and death—whereas the idea of man is imperishable and eternal. It is only by partaking in the nature of these ideas that individual objects exist at all.

Pure Nominalism was the swing of the pendulum of thought to the very opposite extreme ; while Conceptualism was an attempt to hit the happy mean between the two.

Roughly it may be said that the Realists sought for the answer to the question ' What is a Universal ?' in the matter of thought, the Conceptualists in the form, and the Nominalists in the expression.

A full answer to the question ' What is a Universal ?' will bring in something of the three views above given, while avoiding the exaggeration of each. A Universal is

a number of things that are called by the same name; but they would not be called by the same name, unless they fell under the same conception in the mind; nor would they fall under the same conception in the mind, unless there actually existed similar attributes in the several members of a class, causing us to regard them under the same conception and to give them the same name. Universals therefore do exist in nature, and not merely in the mind of man: but their existence is dependent upon individual objects, instead of individual objects depending for their existence upon them. Aristotle saw this very clearly, and marked the distinction between the objects corresponding to the singular and to the common term by calling the former Primary and the latter Secondary Existences. Rosinante and Excalibur are primary, but 'horse' and 'sword' secondary existences.

§ 15. We have seen that the three products of thought are each one stage in advance of the other, the inference being built upon the proposition, as the proposition is built upon the term. Logic therefore naturally divides itself into three parts.

The First Part of Logic deals with the Term;

the Second Part deals with the Proposition;

the Third Part deals with the Inference.

It has been proposed to add a fourth part called Method, which should deal with putting inferences together into a systematic treatise: but not much has been done in that direction. The treatment of Trains of Reasoning really falls under it.

PART I.

OF TERMS.

CHAPTER I.

Of Terms as distinguished from other Words.

§ 16. A TERM is so called because it is the boundary of a proposition. In a proposition we start from a subject and end in a predicate: there is nothing else in a proposition but the copula, which is a mere sign of agreement or disagreement between the two.

§ 17. Hence it appears that the term by its very name indicates that it is arrived at by an analysis of the proposition. It is the judgement or proposition that is the true unit of thought and speech. The proposition as a whole is prior in conception to the terms which are its parts: but the parts must precede the whole in the synthetic order of treatment.

§ 18. All terms are words, but not all words are terms. Only those words are terms which can be used by themselves as the subject or else as the predicate of a proposition. Such words are called Categorematic, which

means literally 'predicable.' A simple test of what words are terms is to ask oneself—'Can I use this word as a predicate?' All adjectives, participles, and verbs are terms, though the verb is a term and something more, since it involves the copula. Words which can only be used along with others to make up terms, e.g. 'of,' 'very,' are called Syncategorematic. If a word does not enter into a proposition at all, it is Acategorematic. Of this nature are conjunctions and interjections, the vocative cases of nouns, and the imperative and optative moods of verbs, which are not used in making statements. From this point of view then a term may be defined as a word or collection of words which can be used by itself as a predicate. It is enough to say this, since any word which can be used as a subject can also be used as a predicate.

§ 19. In grammar every noun is a separate word : but to logic, which is concerned with the thought rather than with the expression, it is indifferent whether a noun, or term (which is merely a noun as used in a proposition), consists of one word or many. The latter are known as 'many-worded names.' In the following passage, taken at random from Butler's *Analogy*—'These several observations, concerning the active principle of virtue and obedience to God's commands, are applicable to passive obedience or resignation to His will'—we find the subject consisting of fourteen words and the predicate of nine. It is the exception rather than the rule to find a predicate which consists of a single word. Many-worded names in English often consist of clauses introduced by the

conjunction 'that,' as 'That letters should be written in strict conformity with nature is true;' often also of a grammatical subject with a dependent clause attached to it, as 'He who fights and runs away.'

§ 20. We have used the word 'term' interchangeably with 'name' or 'noun.'

A Name is a word or collection of words which serves to recall or transmit the idea of a thing either in itself or through some of its attributes.

A name or noun is either a substantive or an adjective.

A Noun Substantive is the name of a thing in itself, i. e. without reference to any particular attribute, e. g. horse, bird.

A Noun Adjective is a name which we add to a thing because it possesses some particular attribute, e. g. lame, grey, flying, singing.

Hence the noun adjective is called in logic an Attributive. It includes the participle which is a verbal adjective.

The verb as such is not recognised by logic, but is analysed into the copula and an attributive, e. g.

The bird sings=The bird is singing.

§ 21. When an attributive appears to be used as a subject, it is owing to a grammatical ellipse. Thus in Latin we say 'Boni sapientes sunt' and in English 'The good are wise,' because it is sufficiently declared by the inflexional form in the one case, and by the usage of the language in the other, that men are signified. It is an

accident of language how far adjectives can be used as subjects. They cease to be logical attributives the moment they are so used.

§ 22. There is a sense in which every word may become categorematic, namely, when it is used simply as a word, to the neglect of its proper meaning. Thus we can say—'Very' is an adverb. 'Very' in this case means 'the word very.' This sense is technically known as the *suppositio materialis* of a word. It was expressed in Greek by using the neuter article with a masculine or feminine noun, e. g. τὸ ἄνθρωπος = the word 'man.'

§ 23. *Univocal and Equivocal Words.*

A Univocal Word has only one meaning, e. g. jampot, cyanide of potassium.

An Equivocal Word has more than one meaning, e. g. gum, Idealism.

Most words are equivocal either to eye or ear or both. Thus 'minute' is equivocal to the eye, 'I' or 'eye' is equivocal to the ear, and 'ear' itself is equivocal both to eye and ear: with the same pronunciation it may mean the organ of hearing or a spike of corn, or as a verb, in old English to plough.

It is usual to have *terms* divided into univocal and equivocal. But an equivocal word is two or more terms, of which the definition in each case would be different. Sometimes 'analogous' is added as a third head to the division. But employment by analogy is one of the ways

in which a word becomes equivocal. The word 'sweet,' for instance, is applied by analogy to things so different in their own nature as a lump of sugar, a young lady, a tune, a poem, and so on. Again, because the head is the highest part of man, the highest part of a stream is called by analogy 'the head'; as also is the highest functionary in a college.

CHAPTER II.

Of the Division of Things.

§ 24. BEFORE entering on the divisions of terms it is necessary to advert for a moment to a division of the things whereof they are names.

By a 'thing' is meant simply an object of thought—whatever one can think about. The word is supposed by some to be etymologically connected with 'think,' as *dinge* in German with *denken,* and *res* in Latin with *reri.*

Things are either Substances or Attributes. Attributes may be subdivided into Qualities and Relations.

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                      Thing
            |           |           |
       Substance              Attribute
                                   |
                          |                 |
                       Quality          Relation.
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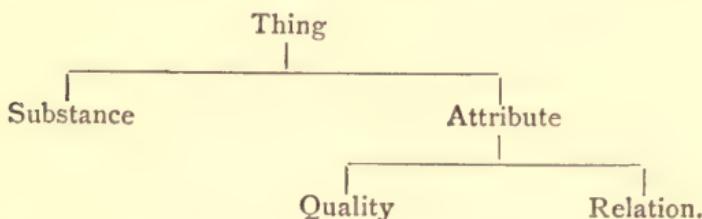
A Substance is a thing which can be conceived to exist by itself.

Substances are either material or immaterial. The body is a material substance; the soul is an immaterial substance.

Substances again are either Primary or Secondary.

'This table' is a primary substance.

'Table' is a secondary substance.

An Attribute is a thing which depends for its existence upon a substance, e. g. greenness, hardness, squareness, which cannot be conceived to exist apart from green, hard, and square substances.

A Quality is an attribute which does not require more than one substance for its existence. The attributes which have just been mentioned are qualities. There might be greenness, hardness, and squareness, if there were nothing in the universe but one green, hard, and square substance.

A Relation is an attribute which requires two or more substances for its existence, e. g. likeness, nearness.

§ 25. When we say that a substance can be conceived to exist by itself, what is meant is that it can be conceived to exist independently of other substances. We do not mean to assert that substances can be conceived to exist out of space and time, nor independently of attributes, nor yet out of relation to a mind perceiving them. The only substance with which we are intuitively acquainted is the Ego. Apart from that, substances, so far as we know them, are only collections of attributes. When therefore we say that substances can be conceived to exist by themselves, whereas attributes are dependent for their existence upon substances, the real meaning of the assertion is this, that it is only certain collections of attributes that can be conceived to exist independently,

whereas single attributes depend for their existence upon others. The colour, smoothness, or hardness of a table cannot be conceived apart from the extension, i. e. the length, breadth, and thickness, whereas the whole cluster of attributes which, as united by the mind, constitutes the table can be conceived to exist altogether independently of other such clusters. We can imagine a table to exist, if the whole material universe were annihilated, and but one mind left to perceive it. Apart from mind however we cannot imagine it, since what we call the attributes of a material substance are no more than the various modes in which we find our minds affected.

The above division of things belongs rather to the domain of metaphysics than of logic : but it is the indispensable basis of the division of terms, to which we now proceed.

CHAPTER III.

Of the Divisions of Terms.

§ 26.

Term		
Subject-term		Division of terms according to their place in thought.
Attributive	Positive, Privative, Negative	
Concrete, Abstract		according to the kind of thing signified.
Singular, Common		according to Quantity in Extension.
Relative, Absolute		according to number of things involved in the name.
Connotative, Non-connotative		according to number of Quantities.

Subject-term and Attributive.

§ 27. A Subject-term is any term which can be used as a subject, e. g. Socrates, wisdom.

An Attributive is a term which signifies the presence or absence of an attribute, e. g. wise, unwise, not-wise.

Attributives can only be used as predicates. They

are contrivances of language whereby we indicate that a subject has a certain attribute. Thus when we say, 'This paper is white,' we indicate that the subject paper possesses the attribute whiteness. Logic however also recognises as attributives terms which signify the non-possession of attributes. 'Not-white' is an attributive equally with white.

Attributives are not names of attributes, but names of the things which possess the attributes, in virtue of our knowledge that they possess them. Thus 'white' is the name of all the things which possess the attribute whiteness, and 'virtuous' is a name, not of the abstract quality, virtue, itself, but of the men and actions which possess it. It is clear that a term can only properly be said to be a name of those things whereof it can be predicated. Now, we cannot intelligibly predicate an attributive of the abstract quality, or qualities, the possession of which it implies. We cannot, for instance, predicate the term 'learned' of the abstract quality of learning: but we may predicate it of the individuals, Varro and Vergil. Attributives then are to be regarded as names, not of the attributes which they imply, but of the things in which those attributes are found.

Attributives however are names of things in a less direct way than that in which subject-terms may be the names of the same things. Attributives are names of things only in predication, whereas subject-terms are names of things in or out of predication. The terms 'horse' and 'Bucephalus' are names of certain things, in

this case animals, whether we make any statement about them or not: but the terms 'swift' and 'fiery' only become names of the same things in virtue of being predicable of them. When we say 'Horses are swift' or 'Bucephalus was fiery,' the terms 'swift' and 'fiery' become names respectively of the same things as 'horse' and 'Bucephalus.' This function of attributives as names in a secondary sense is exactly expressed by the grammatical term 'noun adjective.' An attributive is not directly the name of anything. It is a name added on in virtue of the possession by a given thing of a certain attribute, or, in some cases, the non-possession.

Although attributives cannot be used as subjects, there is nothing to prevent a subject-term from being used as a predicate, and so assuming for the time being the functions of an attributive. When we say 'Socrates was a man,' we convey to the mind the idea of the same attributes which are implied by the attributive 'human.' But those terms only are called attributives which can never be used except as predicates.

This division into Subject-terms and Attributives may be regarded as a division of terms according to their place in thought. Attributives, as we have seen, are essentially predicates, and can only be thought of in relation to the subject, whereas the subject is thought of for its own sake.

§ 28. *Positive, Privative, and Negative.*

This threefold division implies a preceding twofold division—

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                        Term
                         |
        ┌────────────────┴────────────────┐
     Positive                         Non-positive
                                          |
                          ┌───────────────┴───────────────┐
                      Privative                        Negative.
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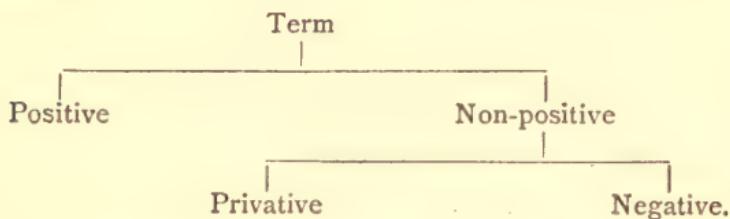
It is only of importance as applied to attributives, though it may be extended to terms generally.

A Positive term signifies the presence of an attribute, e. g. 'wise,' 'full.'

A Negative term signifies merely the absence of an attribute, e. g. 'not-wise,' 'not-full.'

A Privative term signifies the absence of an attribute in a subject capable of possessing it, e. g. 'unwise,' 'empty[1].'

Thus a privative term stands midway in meaning between the other two, being partly positive and partly negative—negative in so far as it indicates the absence of a certain attribute, positive in so far as it implies that the thing which is declared to lack that attribute is of such a nature as to be capable of possessing it. A purely

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[1] The author is glad to find that the extension of meaning which he gave in his *Deductive Logic* to the word 'privative,' has been adopted (not without acknowledgement) by Mr. Welton, Vol. I, p. 83. The name used to be confined to a term signifying the absence of an attribute where it was once possessed or might have been expected to be present, e. g. 'blind.'

negative term conveys to the mind no positive information at all about the nature of the thing of which it is predicated, but leaves us to seek for it among the universe of things which fail to exhibit a given attribute.

A privative term, on the other hand, restricts us within a definite sphere. The term 'empty' restricts us within the sphere of things which are capable of fulness, that is, if the term be taken in its literal sense, things which possess extension in three dimensions.

A positive and a negative term, which have the same matter, must exhaust the universe between them, e. g. 'white' and 'not-white,' since, according to the law of excluded middle, everything must be either one or the other.   To say however that a thing is 'not-white' is merely to say that the term 'white' is inapplicable to it. 'Not-white' may be predicated of things which do not possess extension as well as of those which do.   Such a pair of terms as 'white' and 'not-white,' in their relation to one another, are called Contradictories.

Contrary terms must be distinguished from contradictory [1].   Contrary terms are those which are most opposed under the same head.   Thus 'white' and 'black' are contrary terms, being the most opposed under the same head of colour.   'Virtuous' and 'vicious' again are contraries, being the most opposed under the same head of moral quality.

[1] Called by Cicero 'disparata.'   *De Inv.* I, § 42 'disparatum autem est id, quod ab aliqua re praepositione negationis separatur, hoc modo: sapere et non sapere.'

A positive and a privative term in the same matter are either contraries, e. g. 'wise' and 'unwise' (= foolish), or else the privative includes the contrary, e. g. 'white' and 'unwhite,' which includes 'black' as one of its forms.

Words which are positive in form are often privative in meaning, and vice versâ. This is the case, for instance, with the word 'safe,' which connotes nothing more than the absence of danger. We talk of a thing involving 'positive danger' and of its being 'positively unsafe' to do so and so. 'Unhappy,' on the other hand, signifies the presence of actual misery. Similarly in Latin *inutilis* signifies not merely that there is no benefit to be derived from a thing, but that it is *positively injurious*. All such questions however are for the grammarian or lexicographer, and not for the logician. For the latter it is sufficient to know that corresponding to every term which signifies the presence of some attribute there may be imagined another which indicates the absence of the same attribute, where it might be possessed, and a third which indicates its absence, whether it might be possessed or not.

Negative terms proper are formed by the prefix 'not-' or 'non-,' and are mere figments of logic. We do not in practice require to speak of the whole universe of objects minus those which possess a given attribute or collection of attributes. We have often occasion to speak of things which might be wise and are not, but seldom, if ever, of all things other than wise.

Every privative attributive has, or may have, a corre-

sponding abstract term, and the same is the case with negatives: for the absence of an attribute is itself an attribute. Corresponding to ' empty' there is 'emptiness'; corresponding to ' not-full' there may be imagined the term ' not-fulness.'

The contrary of a given term always involves the contradictory, but it involves positive elements as well. Thus 'black' is ' not-white,' but it is something more besides. Terms which, without being directly contrary, involve a latent contradiction, are called Repugnant, e. g. ' red' and ' blue.' All terms whatever which signify attributes that exclude one another may be called Incompatible.

The preceding division is based on what is known as the Quality of terms, a positive term being said to differ in quality from a non-positive one.

§ 29. *Concrete and Abstract.*

A Concrete Term is the name of a substance, e. g. a man, this chair, the soul, God.

An Abstract Term is the name of an attribute, e. g. whiteness[1], multiplication, act, purpose, explosion.

By a 'concrete thing' is meant an individual substance conceived of with all its attributes about it. The term is not confined to material substances. A spirit conceived

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[1] Since things cannot be spoken of except by their names, there is a constantly recurring source of confusion between the thing itself and the name of it. Take for instance ' whiteness.' The attribute whiteness is a thing, the word ' whiteness ' is a term.

of under personal attributes is as concrete as plum-pudding.

Abstract terms are so called as being arrived at by a process of Abstraction. What is meant by Abstraction will be clear from a single instance. The mind, in contemplating a number of substances, may draw off, or abstract, its attention from all their other characteristics, and fix it only on some point, or points, which they have in common. Thus, in contemplating a number of three-cornered objects, we may draw away our attention from all their other qualities, and fix it exclusively upon their three-corneredness, thus constituting the abstract notion of 'triangle.' Abstraction may be performed equally well in the case of a single object: but the mind would not originally have known on what points to fix its attention except by a comparison of individuals.

Abstraction too may be performed upon attributes as well as substances. Thus, having by abstraction already arrived at the notion of triangle, square, and so on, we may fix our attention upon what these have in common, and so rise to the higher abstraction of 'figure.' As thought becomes more complex, we may have abstraction on abstraction and attributes of attributes. But, however many steps may intervene, attributes may always be traced back to substances at last. For attributes of attributes can mean at bottom nothing but the co-existence of attributes in, or in connexion with, the same substances.

We have said that abstract terms are so called, as being

arrived at by abstraction : but it must not be inferred from
this statement that all terms which are arrived at by ab-
straction are abstract. If this were so, all names would
be abstract except proper names of individual substances.
All common terms, including attributives, are arrived at by
abstraction, but they are not therefore abstract terms.
Those terms only are called abstract which cannot be
applied to substances at all. The terms 'man' and
'human' are names of the same substance of which
Socrates is a name. Humanity is a name only of certain
attributes of that substance, namely those which are shared
by others. All names of concrete things then are concrete,
whether they denote them individually or according to
classes, and whether directly and in themselves, or in-
directly, as possessing some given attribute.

Since things are divided into substances and attributes,
it follows that any term which is not the name of a thing
capable of being conceived to exist by itself must be an
abstract term. Individual substances can alone be con-
ceived to exist by themselves : all their qualities, actions,
passions, and inter-relations, all their states, and all events
with regard to them, presuppose the existence of these
individual substances. All names therefore of such things
as those just enumerated are abstract terms. The term
'action,' for instance, is an abstract term. For how could
there be action without an agent? The term 'act' also
is equally abstract for the same reason. The difference
between 'action' and 'act' is not the difference between
abstract and concrete, but the difference between the name

of a process and the name of the corresponding product. Unless acts can be conceived to exist without agents they are as abstract as the action from which they result.

Since every term must be either abstract or concrete, it may be asked—Are attributives abstract or concrete? The answer of course depends upon whether they are names of substances or names of attributes. But attributives, it must be remembered, are never directly names of anything, in the way that subject-terms are ; they are only names of things in virtue of being predicated of them. Whether an attributive is abstract or concrete depends on the nature of the subject of which it is asserted or denied. When we say 'This man is noble,' the term 'noble' is concrete, as being the name of a substance : but when we say 'This act is noble,' the term 'noble' is abstract, as being the name of an attribute.

The division of terms into Abstract and Concrete is based upon the kind of thing signified. It involves no reference to actual existence. There are imaginary as well as real substances. Logically a centaur is as much a substance as a horse.

§ 30. *Singular and Common.*

A Singular Term is a name which can be applied in the same sense to the one thing only, e.g. ' John,' ' Paris,' ' the capital of France,' 'this pen.'

A Common Term is a name which can be applied in the same sense to a class of things, e. g. ' man,' ' metropolis,' ' pen.' A common term is also called General.

In order that a term may be applied in the same sense to a number of things, it is evident that it must indicate attributes which are common to all of them. The term 'John' is applicable to a number of things, but not in the same sense, as it does not indicate attributes, or, if it does, they are different in each case. When the same proper name is applied to more than one thing, it becomes ambiguous, but not common, being then more than one term.

Common terms are formed, as we have seen already (§ 29), by abstraction, i. e. by withdrawing the attention from the attributes in which individuals differ and concentrating it upon those which they have in common.

A class need not necessarily consist of more than two things. If the sun and moon were the only heavenly bodies in the universe, the word 'heavenly body' would still be a common term, as indicating the attributes which are possessed alike by each.

This being so, it follows that the division of terms into singular and common is as exhaustive as the preceding ones, since a singular term is the name of one thing and a common term of more than one. It is indifferent whether the thing in question be a substance or an attribute; nor does it matter how complex it may be, so long as it is regarded by the mind as one.

Since every term must thus be either singular or common, the members of the preceding divisions must find their place under one or both heads of this one. Subject-terms may fall under either head of singular or

common: but attributives are essentially common terms. Such names as 'green,' 'gentle,' 'incongruous' are applicable, strictly in the same sense, to all the things which possess the attributes which they imply.

Are abstract terms then, it may be asked, singular or common? To this question we reply—That depends upon how they are used. The term 'virtue,' for instance, in one sense, namely, as signifying moral excellence in general, without distinction of kind, is strictly a singular term, as being the name of one attribute: but as applied to different varieties of moral excellence—justice, generosity, gentleness, and so on—it is a common term, as being a name which is applicable in the same sense to a class of attributes. Similarly the term 'colour' in a certain sense signifies one unvarying attribute possessed by bodies, namely, the power of affecting the eye, and in this sense it is a singular term: but as applied to the various ways in which the eye may be affected it is evidently a common term, being equally applicable to red, blue, green, and every other colour. As soon as we begin to apply abstraction to attributes, the higher notion becomes a common term in reference to the lower. By a 'higher notion' is meant one which is formed by a further process of abstraction. The terms 'red,' 'blue,' 'green,' &c., are arrived at by abstraction from physical objects; 'colour' is arrived at by abstraction from them, and contains nothing but what is common to all. It therefore applies in the same sense to each, and is a common term in relation to them.

A practical test as to whether an abstract term, in any given case, is being used as a singular or common term, is to try whether the indefinite article or the sign of the plural can be attached to it. The term 'number,' as the name of a single attribute of things, admits of neither of these adjuncts: but to talk of 'a number' or 'the numbers, two, three, four,' &c., at once marks it as a common term. Similarly the term 'unity' denotes a single attribute, admitting of no shades of distinction : but when a writer begins to speak of 'the unities' he is evidently using the word for a class of things of some kind or other, namely, certain dramatical proprieties of composition.

The division of terms into singular and common is based on their Quantity in Extension. This phrase will be explained presently.

§ 31. *Proper Names and Designations.*

Singular terms may be subdivided into Proper Names and Designations.

A Proper Name is a permanent singular term applicable to a thing in itself, i. e. irrespective of its particular attributes ; a Designation is a singular term devised for the occasion, or applicable to a thing only in so far as it possesses some attribute.

'Homer' is a proper name ; 'this man,' 'the author of the Iliad' are designations.

The number of things, it is clear, is infinite. For, granting that the physical universe consists of a definite

number of atoms—neither one more nor one less—still we are far from having exhausted the possible number of things. All the manifold material objects, which are made up by the various combinations of these atoms, constitute separate objects of thought, or things, and the mind has further an indefinite power of conjoining and dividing these objects, so as to furnish itself with materials of thought, and also of fixing its attention by abstraction upon attributes, so as to regard them as things, apart from the substances to which they belong.

This being so, it is only a very small number of things, which are constantly obtruding themselves upon the mind, that have singular terms permanently set apart to denote them. Human beings, some domestic animals, and divisions of time and place, have proper names assigned to them in most languages, e. g. 'John,' 'Mary,' 'Grip,' 'January,' 'Easter,' 'Belgium,' 'Brussels,' 'the Thames,' 'Ben-Nevis.' Besides these, all abstract terms, when used without reference to lower notions, are of the nature of proper names, being permanently set apart to denote certain special attributes, e. g. 'benevolence,' 'veracity,' 'imagination,' 'indigestibility,' 'retrenchment.'

But the needs of language often require a singular term to denote some thing which has not had a proper name assigned to it. This is effected by taking a common term, and so limiting it as to make it applicable, under the given circumstances, to one thing only. Such a limitation may be effected in English by prefixing a demonstrative or the definite article, or by appending a description, e. g.

'this pen,' 'the sofa,' 'the last rose of summer.'   When a proper name is unknown, or for some reason un-available, recourse may be had to a designation, e. g. 'the honourable member who spoke last but one.'

§ 32. *Collective Terms.*

Collective terms do not fall under the same division with singular and common.   That division is exhaustive, since every term must either be the name of one thing only or of more than one.

A Collective Term is the name of a group of similar things, e. g. flock, regiment, mankind.   Such a term may be either singular or common.   Thus 'mankind' is a singular term which is applicable only to one group, 'regiment' is a common term which is applicable in the same sense to many different groups.

The opposite of Collective is Distributive.

This is not so much a division of terms as a distinction between the uses of the same term. 'Library' is used collectively of the books which compose it, distributively of various collections of books, such as the Bodleian, Queen's Library, and so on.

The distinction between the collective and the dis-tributive use of a term is of importance, because the confusion of the two is a favourite source of fallacy. When it is said 'The plays of Shakespeare cannot be read in a day,' the proposition meets with a different measure of acceptance according as its subject is understood collectively or distributively.

Among singular terms we may distinguish between individual and collective, by the former being meant the name of one object, by the latter the name of several considered as one. 'This key' is an individual term; 'my bunch of keys' is a collective term: but both are singular.

A collective singular term is quite as much the name of one thing as an individual term is, though the thing in question happens to be a group. A group is one thing, if we choose to think of it as one. For the mind, as we have already seen, has an unlimited power of forming its own things, or objects of thought. Thus a particular peak in a mountain chain is as much one thing as the chain itself, though, physically speaking, it is inseparable from it, just as the chain itself is inseparable from the earth's surface. In the same way a necklace is as much one thing as the individual beads which compose it.

We have just seen that a collective term is the name of a group regarded as one thing: but every term which is the name of such a group is not necessarily a collective term. 'London,' for instance, is the name of a group of objects considered as one thing. But 'London' is not a collective term, whereas 'flock,' 'regiment,' and 'senate' are. Wherein then lies the difference? It lies in this—that flock, regiment, and senate are groups composed of objects which are, to a certain extent, similar, whereas London is a group made up of the most dissimilar objects —streets and squares and squalid slums, fine carriages and dirty faces, and so on. In the case of a true collective term all the members of the group will come under some

one common name.   Thus all the members of the group,
flock of sheep, come under the common name 'sheep,'
all the members of the group 'regiment' under the
common name, 'soldier,' and so on.

§ 33.   *Relative and Absolute.*

A Relative term is a name given to a thing with direct
reference to some other thing.

An Absolute term is a name given to a thing without
reference to anything else.

'Hodge' and 'man' are absolute terms.   'Husband,'
'father,' 'shepherd' are relative terms.   'Husband' con-
veys a direct reference to 'wife,' 'father' to 'child,'
'shepherd' to 'sheep.'   Given one term of a relation, the
other is called the correlative, e. g. 'subject,' is the cor-
relative of 'ruler,' and conversely 'ruler' of 'subject.'   The
two terms are also spoken of as a pair of correlatives.

The distinction between relative and absolute applies
to attributives as well as subject-terms.   'Greater,' 'near,'
'like,' are instances of attributives which everyone would
recognise as relative.

A relation, it will be remembered, is a kind of attribute,
differing from a quality in that it necessarily involves
more substances than one.   Every relation is at bottom
a fact, or series of facts, in which two or more substances
play a part.   A relative term connotes this fact or facts
from the point of view of one of the substances, its
correlative from that of the other.   Thus 'ruler' and
'subject' imply the same set of facts, looked at from

opposite points of view. The series of facts itself, regarded from either side, is denoted by the corresponding abstract terms, 'rule' and 'subjection.'

The names of relations are not themselves relative terms. 'Rule' and 'subjection' are not two different things which receive their names with reference to one another, but are the same fact looked at from opposite points of view. 'Ruler' and 'subject,' on the other hand, are names of two distinct substances, but each involving a reference to the other. This division then may be said to be based on the number of things involved in the name.

§ 34. *Connotative and Non-connotative.*

A Connotative term is one which has Extension and Intension distinct from one another.

A Non-connotative term is one which has Extension only, or in which Extension and Intension coincide.

This division of terms is based on their possession of one quantity or two. It will become intelligible when we have explained what is meant by the quantity of terms.

§ 35. *The Quantity of Terms.*

A term is possessed of Quantity in two ways—

   (1) in Extension;
   (2) in Intension.

The Extension of a term is the number of things to which it applies.

The Intension of a term is the number of attributes which it implies.

To take an example, the term 'man' applies to certain things, namely, all the members of the human race that have been, are, or ever will be: this is its quantity in extension. But the term man has also a certain meaning, and implies certain attributes—rationality, animality, and a definite bodily shape, together with others which flow from or are found conjoined with these: the sum of these attributes constitutes its quantity in intension.

The distinction between the two kinds of quantity possessed by a term is also conveyed by a variety of expressions which are here appended.

Extension = breadth = compass = application = denotation.

Intension = depth = comprehension = implication = connotation.

Of these various expressions, 'application' and 'implication' have the advantage of most clearly conveying their own meaning. 'Extension' and 'intension' however are more usual. 'Connotation' is not quite exact as a synonym for intension. It is the intension of a term which possesses a distinct extension. A word must already note one thing before it can be said to *con*note another. We say then that a term connotes attributes when it implies certain attributes at the same time that it applies to certain things distinct therefrom [1].

[1] By the Schoolmen 'connotative' was sometimes used in the same sense in which we have used 'attributive,' for a word which directly signifies the presence of an attribute and indirectly applies to a subject. In this sense it was the subject which was said to be connoted and not the attribute.

The subject-term ' man,' and its corresponding attributive, ' human,' have both extension and intension, distinct from one another. They are therefore connotative. But the abstract term, ' humanity,' denotes the very collection of attributes which was before connoted by the concrete terms, ' man ' and ' human.' In this case, therefore, extension and intension coincide, and the term is non-connotative.

The above remark must be understood to be limited to abstract terms in their singular sense. When abstract terms become common terms through the classification of attributes, they become also connotative. Thus ' colour ' denotes ' red,' ' blue,' ' green,' &c., and connotes what is common to those attributes.

Even when used in a singular sense an abstract term may be said to possess connotation, if we distinguish between the attribute to which the name applies and some other attribute associated therewith by experience or necessity. Thus ' whiteness ' denotes a certain attribute and connotes hurtfulness to the eyes and also extension.

Since all terms are names of things, whether substances or attributes, it is clear that all terms must possess extension, though the extension of singular terms is the narrowest possible, as being confined to one thing.

Are there then any terms which possess no intension? To ask this is to ask—Are there any terms which have absolutely no meaning? It is often said that proper names are devoid of meaning, and the remark is, in

a certain sense, true. When we call a being by the name 'man,' we do so because that being possesses human attributes, but when we call the same being by the name, 'John,' we do not mean to indicate the presence of any Johannine attributes. We simply wish to distinguish that being, in thought and language, from other beings of the same kind. Roughly speaking therefore proper names are devoid of meaning or intension. But no name can be entirely devoid of meaning. For, even setting aside the fact, which is not universally true, that proper names indicate the sex of the owner, the mere act of giving a name to a thing implies at least that the thing exists, whether in fact or thought; it implies what we may call 'thinghood': so that every term must carry with it some small amount of intension.

From another point of view however proper names possess more intension than any other terms. For when we know a person, his name calls up to our minds all the individual attributes with which we are familiar, and these must be far more numerous than the attributes which are conveyed by any common term which can be applied to him. Thus the name 'John' means more to a person who knows him than 'attorney,' 'conservative,' 'scamp,' or 'vestry-man,' or any other term which may happen to apply to him. This however is the acquired intension of a term, and must be distinguished from the original intension. 'Edwin' and 'Angelina' at first signify no more than 'he' or 'she': later on they may acquire a world of meaning. While then the *acquired* intension of a proper

name may be larger than that of any other term, its *original* intension may be next to nothing.

Hitherto we have been speaking only of christening-names, but it is evident that family names have a certain amount of connotation from the first. For when we dub John with the additional appellation of Smith, we do not give this second name as a mere individual mark, but intend thereby to indicate a relationship to other persons. The amount of connotation that can be conveyed by proper names is very noticeable in the Latin language. Let us take for an example the full name of a distinguished Roman—Publius Cornelius Scipio Aemilianus Africanus minor. Here it is only the praenomen, Publius, that can be said to be a mere individual mark, and even this distinctly indicates the sex of the owner. The nomen proper, Cornelius, declares the wearer of it to belong to the illustrious gens Cornelia. The cognomen, Scipio, further specifies him as a member of a distinguished family in that gens. The agnomen adoptivum indicates his transference by adoption from one gens to another. The second agnomen recalls the fact of his victory over the Carthaginians, while the addition of the word 'minor' distinguishes him from the former wearer of the same title. The name, instead of being devoid of meaning, is a chapter of history in itself[1]. Homeric epithets, such as

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[1] Mr. Welton (Vol. I, p. 66) does not agree here. A Roman name, according to him, very strongly *suggests*, but does not *imply* attributes. But was not the name Gaius Julius Caesar given to a particular infant to indicate his possession of certain

'The Cloud-compeller,' 'The Earth-shaker' are instances of intensive proper names. Many of our own family names are obviously connotative in their origin, implying either some personal peculiarity, e. g. Armstrong, Cruikshank, Courteney ; or the employment, trade, or calling of the original bearer of the name, Smith, Carpenter, Baker, Clark, Leach, Archer, and so on ; or else his abode, domain, or nationality, as De Caen, De Montmorency, French, Langley ; or simply the fact of descent from some presumably more noteworthy parent, as Jackson, Thomson, Fitzgerald, O'Connor, Macdonald, Apjohn, Price, Davids, &c. The question however whether a term is connotative or not has to be decided, not by its origin, but by its use. We have seen that there are some proper names which, in a rough sense, may be said to possess no intension.

The other kind of singular terms, namely designations, (§ 31) are obviously connotative. We cannot employ even the simplest of them without conveying more or less information about the qualities of the thing which they are used to denote. When, for instance, we say 'this table,' 'this book,' we indicate the proximity to the speaker of the object in question. Other designations have a higher degree of intension, as when we say 'the

attributes—Gaius to indicate his sex, Julius his clan, Caesar his family? If a proper name is never to be allowed to have a meaning, Mr. Welton ought rather to contend that a Roman proper name was not in the strict sense a proper name at all, since it might apply in the same sense generation after generation to any one of a particular sex, clan, and family.

present prime minister of England,' 'the honourable member who brought forward this motion to-night.' Such terms have a good deal of significance in themselves, apart from any knowledge we may happen to possess of the individuals they denote.

We have seen that, speaking quite strictly, there are no terms which are non-connotative: but, for practical purposes, we may apply the expression to proper names, on the ground that they possess no intension, and to singular abstract terms on the ground that their extension and intension coincide. In the latter case it is indifferent whether we call the quantity extension or intension. Only we cannot call it 'connotation,' because that implies two quantities distinct from one another. A term must already denote a subject before it can be said to connote its attributes.

§ 36. *Names of Homogeneous Substances.*

The terms which the student will probably find most difficult to classify are names of homogeneous substances, such as gold, air, chalk, milk, sugar, nitrogen, snow, coal. Such terms have been variously called Substantial, Material, and Homogeneous—not very happily however, for these names are applicable to the things denoted by the terms, not to the terms themselves.

Names of this sort present some of the characteristics of abstract terms. For—

(1) it is the intensive force that is uppermost in our minds when we use them ;

(2) they do not as a rule admit of the plural or of the indefinite article.

But although they are like abstract terms, they are plainly not abstract, so that there is no room for controversy on that point. Are they then singular or common? According to Professor Bain[1] they are singular and collective; according to Mr. Keynes they are common. Now such a question cannot be argued in the abstract. We must take a particular use of some one of these words. When we say 'Oil is lighter than water,' we do not wish to assert that the entire collection of oil in the world is lighter than the entire collection of water, which is obvious from there being so much less of it; rather we mean that the specific gravity of any portion of oil you like to take is less than that of an equal portion of water. Now oil in general stands to this or that specimen of oil in the relation of a class to the individuals contained under it. If 'oil' is thus found to be a common term even when used as a subject, much more will it be found to be so when it is used as a predicate. For in this particular case we *can* use the indefinite article and the sign of the

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[1] Bain, *Logic, Deduction,* pp. 48, 49—'Names of Material—earth, stone, salt, mercury, water, flame—are singular. They each denote the entire collection of one species of material.' The term oil is classified by Jevons (*Studies in Deductive Logic,* ch. i, § 21) as collective. Mr. Welton (*Manual of Logic,* Vol. I, p. 61) follows Dr. Venn in regarding 'substantial terms' as a peculiar kind of collective terms. Minto (*Logic, Inductive and Deductive,* p. 59) argues in the same direction. For the opposite view see Keynes (*Formal Logic,* third edit. pp. 9, 10, 12).

plural. We can say 'This substance is an oil,' or 'All these substances are oils.' But let us take instances in which it is not natural to speak thus. We cannot say 'This is a milk' or 'This is a sugar,' in the same way as we say 'This is a cow' or 'This is a plant.' But it does not follow that 'milk' and 'sugar' are not common terms. 'The milk' is, of course, a singular term, but it is individual rather than collective, since the continuity of the parts of milk prevents us from regarding them as separate things: but 'milk,' like 'cow,' is applicable distributively to any primary substance which possesses certain attributes, and is therefore as much a common term as 'cow[1].' The absence of the sign of the plural in a word, and the impropriety of using the indefinite article with it, are sure indications that the attention is concentrated upon its intension. Whenever the extension becomes of importance, the distinction between singular and plural reappears. A waiter will understand you, if you order 'three milks' or 'an ice.'

The remark just made as to extension applies not only to these concrete terms, but also to the abstract terms which they resemble (cp. § 29).

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[1] Aristotle (*Top.* I. 7, § 3) says that water from the same well differs from the ordinary case of individuals of the same species only in the higher degree of resemblance.

CHAPTER IV.

Of the Law of Inverse Variation of Extension and Intension.

§ 37. In a series of terms which fall under one another as the extension decreases, the intension increases, and vice versâ. Take for instance the following series—

Thing
|
Substance
|
Matter
|
Organism
|
Animal
|
Vertebrate
|
Mammal
|
Ruminant
|
Sheep
|
This sheep.
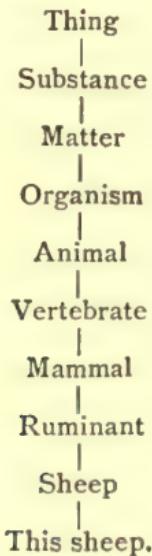
Here the term at the top possesses the widest possible extension, since it applies to everything. But at the same time it possesses the least possible amount of intension, implying nothing more than mere existence, whether in fact or thought. On the other hand, the term at the bottom possesses the greatest amount of intension, since it implies all the attributes of an individual superadded

to those of the class to which it belongs : but its extension is the narrowest possible, being limited to one thing.

At each step in the descent from the term at the top, which is called the *Summum genus*, to the individual, we decrease the extension by increasing the intension. Thus by adding on to the bare notion of a thing the idea of independent existence, we descend to the term ' substance.' This process is known as Determination, or Specialisation.

Again, by withdrawing our attention from the individual characteristics of a particular sheep, and fixing it upon those which are common to it with other animals of the same kind, we arrive at the common term, ' sheep.' Here we have increased the extension by decreasing the intension. This process is known as Generalisation.

Generalisation implies abstraction, but we may have abstraction without generalisation.

The following example is useful, as illustrating to the eye how a decrease of extension is accompanied by an increase of intension. At each step of the descent here we visibly tack on a fresh attribute [1].

<div align="center">

Ship
|
Steam-ship
|
Screw steam-ship
|
Iron screw steam-ship
|
British iron screw steam-ship.

</div>
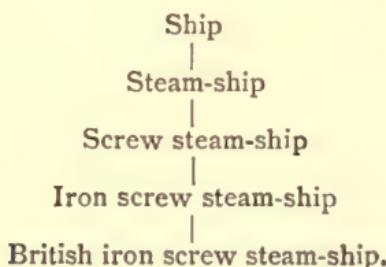
[1] This example is borrowed from Professor Jevons.

Could we see the classes denoted by the names, the pyramid would be exactly inverted.

The law of inverse variation of extension and intension must of course be confined to the inter-relations of a series of terms of which each can be predicated of the other until we arrive at the bottom of the scale. It is not meant to apply to the extension and intension of the same term. The increase of population does not diminish the meaning of ' baby.'

PART II.

OF PROPOSITIONS.

CHAPTER I.

Of the Proposition as distinguished from other Sentences.

§ 38. ALL propositions are sentences, but not all sentences are propositions.

Sentences may be used for a variety of purposes—

(1) To ask a question;

(2) To give an order;

(3) To express a feeling;

(4) To make a statement.

These various uses give rise respectively to

(1) The Interrogative;

(2) The Imperative;

(3) The Exclamatory;   ⎫
                                        ⎬ Sentence.
(4) The Enunciative ⎰ Indicative   ⎭
                     ⎱ Potential

It is with the last of these only that logic is concerned.

The proposition, therefore, corresponds to the Indicative and Potential sentences of grammar. For it must

be borne in mind that logic recognises no difference between a statement of fact and a supposition. 'It may rain to-morrow' is as much a proposition as 'It is raining now.'

§ 39. Leaving the grammatical aspect of the proposition, we must now consider it from the purely logical point of view.

A proposition is a judgement expressed in words[1]; and a judgement is a direct comparison between two concepts.

The same thing may be expressed more briefly by saying that a proposition is a direct comparison between two terms.

We say 'direct comparison,' because the syllogism also may be described as a comparison between two terms: but in the syllogism the two terms are compared indirectly, or by means of a third term.

§ 40. A proposition consists of three parts—

> Subject,
> Predicate,
> Copula.

The Subject is that of which something is said.

The Predicate is that which is said of the subject.

The Copula is the sign of agreement or disagreement between the two.

It is apparent from the definitions that the subject is thought of for its own sake, and the predicate for the sake of the subject.

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[1] Plat. *Phil.* 38 E καὶ λόγος δὴ γέγονεν οὕτως ὃ τότε δόξαν ἐκαλοῦμεν.

CHAPTER II.

Of the Copula.

§ 41. THERE are two kinds of copula, one for affirmative and one for negative statements.

Materially the copula is expressed by some part of the verb 'to be,' with or without the negative, or else is wrapped up in some inflexional form of a verb.

The material form of the copula is an accident of language, and a matter of indifference to logic. 'The kettle boils' is as logical a form of expression as 'The kettle is boiling.' For it must be remembered that the word 'is' here is a mere sign of agreement between the two terms, and conveys no notion of actual existence. It may be used indeed with equal propriety to express non-existence, as when St. Paul says 'An idol is nothing.'

When the verb 'to be' expresses existence in fact, it is known in grammar as 'the substantive verb.' In this use it is predicate as well as copula, as when we say 'God is,' which may be analysed, if we please, into 'God is existent.'

§ 42. We have laid down above that there are two kinds of copula, affirmative and negative : but some logicians have maintained that the copula is always affirmative.

What then, it may be asked, on this view, is the meaning of negative propositions? To which the answer is, that a negative proposition asserts an agreement between the subject and a negative term. When, for instance, we say 'The whale is not a fish,' this would be interpreted to mean 'The whale is a not-fish.'

Undoubtedly any negative proposition may be exhibited in an affirmative form, since, by the law of excluded middle, given a pair of contradictory terms, wherever the one can be asserted, the other can be denied, and vice versâ. We shall find later on that this principle gives rise to one of the forms of immediate inference. The only question then can be, Which is the more natural and legitimate form of expression? It seems simpler to suppose that we assert the agreement of 'whale' with 'not-fish' by implication only, and that what we directly do is to predicate a disagreement between 'whale' and the positive attributes connoted by 'fish.' For since 'not-fish' must apply to every conceivable object of thought except those which fall under the positive term 'fish,' to say that a whale is a 'not-fish' is to say that we have still to search for 'whale' throughout the whole universe of being, minus a limited portion; which is only a more clumsy way of saying that it is not to be found in that portion.

Again, the term 'not-fish' must be understood either in its intension or in its extension. If it be understood in its intension, what it connotes is simply the absence of the positive qualities which constitute a fish, a meaning

which is equally conveyed by the negative form of proposition. We gain nothing in simplicity by thus confounding assertion with denial. If, on the other hand, it is to be taken in extension, this involves the awkwardness of supposing that the predicative power of a term resides in its extensive capacity.

We therefore recognise predication as being of two kinds—affirmation and negation—corresponding to which there are two forms of copula.

§ 43. On the other hand, other logicians have maintained that there are many kinds of copula, since the copula must vary according to the various degrees of probability with which we can assert or deny a predicate of a subject. This view is technically known as the doctrine of

The Modality of the Copula.

It may plausibly be maintained that the division of propositions into affirmative and negative is not an exhaustive one, since the result of an act of judgement is not always to lead the mind to a clear assertion or a clear denial, but to leave it in more or less doubt as to whether the predicate applies to the subject or not. Instead of saying simply S is P, or S is not P, we may be led to one of the following forms of proposition—

S is possibly P.

S is probably P.

S is certainly P.

The adverbial expression which thus appears to qualify the copula is known as 'the mode.'

When we say 'The accused may be guilty' we have a proposition of very different force from 'The accused is guilty,' and yet the terms appear to be the same. Wherein then does the difference lie? 'In the copula' is the obvious answer. We seem therefore driven to admit that there are as many different kinds of copula as there are different degrees of assurance with which a statement may be made.

But there is another way in which modal propositions may be regarded. Instead of the mode being attached to the copula, it may be considered as itself constituting the predicate, so that the above propositions would be analysed thus—

> That S is P is possible.
> That S is P is probable.
> That S is P is certain.

The subject here is itself a proposition, of which we predicate various degrees of probability. In this way the division of propositions into affirmative and negative is rendered exhaustive. For wherever before we had a doubtful assertion, we have now an assertion of doubtfulness.

If degrees of probability can thus be eliminated from the copula, much more so can expressions of time, which may always be regarded as forming part of the predicate. 'The sun will rise to-morrow' may be analysed into 'The sun is going to rise to-morrow.' In either case the tense belongs equally to the predicate. It is often an awkward task so to analyse propositions relative to past or future

time as to bring out the copula under the form 'is' or 'is not': but fortunately there is no necessity for so doing, since, as has been said before (§ 41), the material form of copula is a matter of indifference to logic. Indeed in affirmative propositions the mere juxtaposition of the subject and predicate is often sufficient to indicate their agreement, e.g. 'Most haste, worst speed,' χαλεπὰ τὰ καλά. It is because all propositions are not affirmative that we require a copula at all. Moreover the awkwardness of expression just alluded to is a mere accident of language. In Latin we may say with equal propriety 'Sol orietur cras' or 'Sol est oriturus cras'; while past time may also be expressed in the analytic form in the case of deponent verbs, as 'Caesar est in Galliam profectus'—'Caesar is gone into Gaul.'

The copula then may always be regarded as pure, that is, as indicating mere agreement or disagreement between the two terms of the proposition.

CHAPTER III.

Of the Divisions of Propositions.

§ 44.

Proposition
- True / False } Division according to Quality of Matter.
- Simple
- Complex { Conjunctive / Disjunctive } according to Form.
- Verbal / Real } according to Matter.
- Definite { Universal / Particular } according to Quantity.
- Indefinite
- Affirmative / Negative } according to Quality of Form.

True and False.

§ 45. A proposition is true when its terms are put together or separated in speech in the same way as the things of which they are names are put together or separated in nature. When this is not the case the proposition is false[1].

All truth and falsehood is confined to judgements and propositions. A concept may be real or imaginary, but it is not true or false. 'Horse' is a real concept, 'centaur' is an imaginary one. But 'centaur' by itself is neither

[1] Plat. *Rep.* 413 A ἢ οὐ τὸ τὰ ὄντα δοξάζειν ἀληθεύειν δοκεῖ σοι εἶναι; Ar. *Met.* Θ, 10, § 1 ἀληθεύει μὲν ὁ τὸ διῃρημένον οἰόμενος διαιρεῖσθαι καὶ τὸ συγκείμενον συγκεῖσθαι.

true nor false. It is not until I form some judgement or enunciate some proposition that truth or falsehood emerges, e.g. 'I met a Centaur yesterday, as I came round Carfax.' Again, an inference may be valid or invalid, but it is not true or false. We may have a perfectly valid inference which starts from false premisses and lands us in a false conclusion.

To be true or false is the Quality of the Matter of a proposition; to be affirmative or negative is the Quality of the Form.

Simple and Complex.

§ 46. A Simple proposition makes a statement directly—

<div align="center">S is P.</div>

<div align="center">S is not P.</div>

It is also called Categorical.

A Complex proposition makes a statement subject to some condition.

Hence the complex proposition is also known as Conditional.

Every complex proposition consists of two parts—

(1) Antecedent,

(2) Consequent.

The Antecedent is the condition under which the statement is made.

The Consequent is the statement made subject to it.

The antecedent is so called because it precedes the consequent in the order of thought, but it may either precede or follow it in the order of language.

Thus we may say indifferently—'If the wind drops we shall have rain,' or 'We shall have rain, if the wind drops.'

There are two kinds of complex proposition—

(1) Conjunctive or Hypothetical.

If A is B, C is D.

If the wind drops, we shall have rain.

(2) Disjunctive.

Either A is B or C is D.

Either rain must come or the crops will be spoilt.

The conjunctive proposition may also assume the forms—

If A is, B is.

If A is B, A is C.

If A is C, B is C.

The disjunctive proposition may also assume the forms—

Either A is or B is.

A is either B or C.

Either A or B is C.

As the double nomenclature may cause some confusion, a scheme is appended.
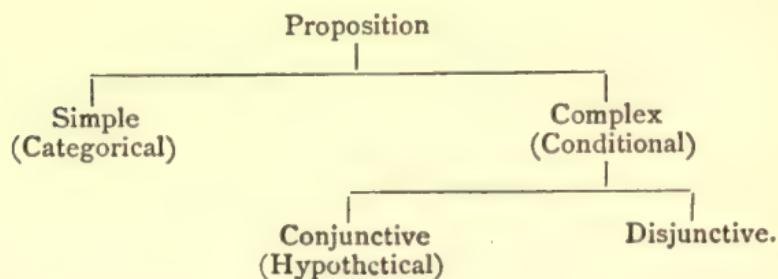
In the older writers the places of Hypothetical and

Conditional are interchanged, Hypothetical being the genus and Conditional the species. The one word is merely a translation of the other.

In the conjunctive proposition the truth of the antecedent involves the truth of the consequent.

In the disjunctive proposition the falsity of the antecedent involves the truth of the consequent.

The disjunctive proposition is a conjunctive with a negative antecedent.

Either A is B or C is D = If A is not B, C is D.

These points will be made clearer by illustration. When, for instance, we say 'If the sky falls, we shall catch larks,' what is it that we really mean to assert? Not that the sky will fall, and not that we shall catch larks, but a certain connexion between the two, namely, that the truth of the antecedent involves the truth of the consequent. Hence this form of proposition is aptly called 'conjunctive,' because in it the truth of the consequent is conjoined to the truth of the antecedent.

Again, when we say 'Jones is either a knave or a fool,' what is really meant to be asserted is—'If you do not find Jones to be a knave, you may be sure that he is a fool.' Here it is the falsity of the antecedent which involves the truth of the consequent; and the proposition is known as 'disjunctive,' because the truth of the consequent is disjoined from the truth of the antecedent.

The complex proposition then is a proposition about propositions, showing the relation in which they stand to one another as regards truth and falsity. It may be

condensed into a simple proposition of which the terms are themselves propositions.

> If the sky falls, we shall catch larks =
>
> Sky-falling is lark-catching.

And generally—

> If A is B, C is D =
>
> Cases of A being B are cases of C being D.

Which for convenience we shall express as follows—

> AB is CD.

Ultimately then every proposition is reducible to the simple form—S is, or is not, P.

Hence this division turns upon the form of expression, and may be said to be founded on the simplicity or complexity of the terms employed in a proposition.

Compound Sentences.

§ 47. A Complex proposition is to be distinguished from a Compound Sentence[1].

The latter is merely a contrivance of language for abbreviating expression, in which several distinct state-

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[1] This was called by the Stoics ἀξίωμα συμπεπλεγμένον. They divided ἀξιώματα into ἁπλᾶ and οὐχ ἁπλᾶ, and among the latter distinguished the συνημμένον (conjunctive) as 'If it is day, it is light,' the παρασυνημμένον, as 'As it is day, it is light,' the συμπεπλεγμένον, as 'It is both day and light,' the διεζευγμένον (disjunctive), as 'Either it is day or night,' and the αἰτιῶδες (causative), as 'Because it is day, it is light.' The Antecedent was called by them τὸ ἀρχόμενον or τὸ ἡγούμενον and the Consequent τὸ λῆγον. See Diog. Laert. VII, §§ 68–73.

ments are combined into a single sentence. In a complex proposition there appears to be more than one subject or predicate or both, while in reality there is only a single statement; and this statement refers, as we have seen, to a certain connexion between two propositions. Thus when we say 'Either the Carthaginians were of Semitic origin or argument from language is of no value in ethnology,' we have two propositions only in appearance.

In a compound sentence on the other hand there is logically, and not merely grammatically, more than one subject or predicate or both. Thus when we say 'The Jews and Carthaginians were Semitic peoples and spoke a Semitic language,' we have four propositions compressed into a single sentence for the sake of brevity.

Verbal and Real.

§ 48. A Verbal proposition gives the definition or part of the definition of the subject.

A Real proposition states some fact about the subject which is not contained in its definition.

To say that a triangle is a figure or that it has three sides or that it is a three-sided figure are all verbal propositions. To say that it has three angles is a real proposition; for this fact, although equally obvious with the former and directly indicated by the etymology, is no part of the concept 'figure contained by three sides.'

It will be seen that the distinction between verbal and real propositions assumes a knowledge of the precise meaning of terms, that is to say, a knowledge of definitions.

To a person who does not know the meaning of terms a verbal proposition will convey as much information as a real one. To say 'The sun is in mid-heaven at noon,' though a merely verbal proposition, will convey information to a person who is being taught to attach a meaning to the word 'noon.' We use so many terms without knowing their meaning, that a merely verbal proposition appears a revelation to many minds. Thus there are people who are surprised to hear that the lion is a cat, though in its definition 'lion' is referred to the class 'cat.' The reason of this is that we know material objects far better in their extension than in their intension, that is to say, we know what things a name applies to without knowing the attributes which those things possess in common.

There is nothing in the mere look of a proposition to inform us whether it is verbal or real; the difference is wholly relative to, and constituted by, the definition of the subject. When we have accepted as the definition of a triangle that it is 'a figure contained by three sides,' the statement of the further fact that it has three angles becomes a real proposition. Again, the proposition 'Man is progressive' is a real proposition. For though his progressiveness is a consequence of his rationality, still there is no actual reference to progressive-

ness contained in the usually accepted definition, ' Man is a rational animal.'

If we were to admit, under the term ' verbal proposition,' all statements which, though not actually contained in the definition of the subject, are implied by it, the whole body of necessary truth would have to be pronounced merely verbal, and the most penetrating conclusions of mathematicians set down as only another way of stating the simplest axioms from which they started. For the propositions of which necessary truth is composed are so linked together that, given one, the rest can always follow. But necessary truth, which is arrived at *a priori*, that is, by the mind's own working, is quite as real as contingent truth, which is arrived at *a posteriori*, or by the teachings of experience, in other words, through our own senses or those of others.

The process by which real truth, which is other than deductive, is arrived at *a priori* is known as Intuition. E.g. The mind sees that a figure with three sides cannot but have three angles.

Only such propositions then must be considered verbal as state facts expressly mentioned in the definition.

The same distinction as between verbal and real propositions is conveyed by the expressions 'Analytical' and 'Synthetical,' or 'Explicative' and 'Ampliative,' judgements.

A verbal proposition is called analytical, as breaking up the subject into its component notions.

A real proposition is called synthetical, as attaching some new notion to the subject.

Among the scholastic logicians verbal propositions were known as 'Essential,' because what was stated in the definition was considered to be of the essence of the subject, while real propositions were known as 'Accidental.'

Definite and Indefinite.

§ 49. A Definite proposition is one of which the quantity is determined as either Universal or Particular.

An Indefinite proposition is one of which the quantity is left undetermined.

The quantity of a proposition is determined by the quantity in extension of its subject.

A Universal proposition is one in which the subject is explicitly used in its whole extent.

A Particular proposition is one in which the subject is explicitly used in part of its extent.

An Indefinite proposition is one in which it is not apparent whether the subject is used in its whole extent or not.

All men are liars.	Universal.	} Definite.
Some men are liars.	Particular.	
Men are liars.		Indefinite.

When the value of some quantity varies, it is a rule of prudent calculation that it should not be put at its highest. Hence the indefinite proposition ranks as particular. Sometimes however the nature of the matter is such as to determine it as universal, e.g. when we say

'Triangles have their three angles equal to two right angles,' it is known that all triangles are meant. But on the other hand when we say 'Apprentices are idle,' we are not supposed to mean that there is no such thing as an industrious apprentice, but only that idleness is the prevailing characteristic of the class.

The word 'some' in conversation is often taken to mean 'some only.' If we were to inform a person that 'Some cows chew the cud,' he might reasonably infer our belief that there are some that do not: but this implication never attaches to the word in logic. When we say 'Some men are sinners,' we must not be taken to be denying that all are included under sin, but we may have reason to know that the proposition holds true of some men, e. g. ourselves, and not wish to go beyond our evidence in the statement. The definitely particular proposition then does not, any more than the indefinite, exclude the possibility of the universal being true.

The definitely particular proposition, be it remembered, means one in which a statement is explicitly made of part of the subject—it means that and nothing more. Whether this part is itself determined or undetermined does not enter into the question. Here again we are perplexed by the ambiguity of the word 'some,' which is sometimes equivalent to the Latin *quidam*, sometimes to *aliquis* : but this difference, though important in other respects, does not affect the definiteness of the proposition.

The division of propositions into universal and particular is based upon their Quantity.

Singular and General.

§ 50. Universal propositions may be subdivided into Singular and General.

A Singular proposition is one of which the subject is a singular term.

A General proposition is one of which the subject is a common term taken in its whole extent.

Singular $\begin{cases} \text{John is a man.} \\ \text{Virtue is beautiful.} \end{cases}$

General $\begin{cases} \text{All men are mortal.} \\ \text{All virtues are praiseworthy.} \end{cases}$

A singular proposition is necessarily universal[1] in the sense in which that term has been defined, because the subject, if used at all, must be used in its whole extent.

The most usual signs of generality are the words 'all,'

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[1] Father Clarke (*Logic*, p. 274) maintains that the singular proposition is the extreme form of the particular. Materially this cannot be denied. But it is a case where extremes meet, and the singular proposition coincides formally with the universal. In the proposition 'Caesar is famous in history' the predicate 'is asserted of each and all the individuals comprised under the subject' (*Ibid.* p. 272). It is otherwise in such a proposition as 'This stone is valuable.' There it may be maintained that the subject is the class-name 'stone,' and that 'this' forms no part of it any more than the 'some' forms part of the subject of a particular proposition ; for, if it did, it would no longer be true to say that the predicate does not apply to the whole of the subject.

'every,' 'each,' in affirmative, and the words 'no,' 'none,' 'not one,' &c., in negative propositions.

There is another use of the word 'general,' in which a general truth or a general proposition means one that holds true in the main, though not absolutely without exception. It is a general truth, for instance, in this sense, that men will buy in the cheapest market.

Affirmative and Negative.

§ 51. This division is based on the Quality of the Form, which is commonly called simply the Quality of a proposition.

It should be noticed that the quality of a proposition is not affected by the quality of its terms, but is determined by the copula. All not-S is not-P is an affirmative proposition.

The Fourfold Division of Propositions.

§ 52. By combining the division according to quantity with that according to quality we obtain four kinds of propositions, namely,

Universal Affirmative. All S is P. (A)
Universal Negative. No S is P. (E)
Particular Affirmative. Some S is P. (I)
Particular Negative. Some S is not P. (O)

Indefinite propositions have to be quantified before they can be brought under this classification. This is a matter of convenience, and does not imply any slur

on the character of the indefinite proposition, which is as legitimate a form of expression as any other.   Its function is to make a statement without raising the question of quantity.   This is done when the subject is used strongly in its intensive capacity.   Thus when we say ' Men are fallible,' what we wish to lay stress on is that the attributes of humanity involve that of fallibility.   If any one asks the awkward question whether they always and necessarily involve it, he forces us to quantify our subject.

When taken with this proviso, the above classification is exhaustive.   Every proposition, no matter what its form may be, must fall under one or other of these four heads.   For every proposition must be either universal or particular, in the sense that the subject must either be known to be used in its whole extent or not ; and any proposition, whether universal or particular, must be either affirmative or negative, for by denying modality of the copula we have excluded everything intermediate between downright assertion and denial.   This classification therefore may be regarded as a Procrustes' bed, into which every proposition is bound to fit at its proper peril.

These four kinds of propositions are represented respectively by the symbols A, E, I, O.

The vowels A and I, which denote the two affirmatives, occur in the Latin words *affirmo* and *aio* ; E and O, which denote the two negatives, occur in the Latin word *nego.*

Extensive and Intensive Propositions.

§ 53. The same proposition may be read in extension
or in intension or partly in one and partly in the other.
Thus when we say ' Cows are ruminants,' the proposition
may be understood to mean that the smaller class ' cow '
is contained in the larger class ' ruminant,' or that the
attributes which make up the concept ' cow ' contain, or
are accompanied by, the attribute of chewing the cud. In
the former case the proposition is read wholly in exten-
sion, in the latter case wholly in intension. As a rule the
natural mode of interpretation is a mixed one, the subject
being used in extension and the predicate in intension.
We have just seen that in the indefinite proposition the
attention is concentrated on the intensive capacity of
a term. But in the case of a subject the extension can
never be wholly lost sight of. When we talk of ' cows '
we mean things which possess certain attributes called
bovine, with which we are all of us familiar, though we
might be puzzled as to how to express them. When we
predicate ' ruminants ' of ' cows,' we mean that among
their strictly bovine attributes, or superadded thereto,
there will be found the attribute of chewing the cud.
The extensive force of the subject is marked in language
by the fact that the subject must always be a substantive.
The predicate on the other hand is generally an adjective,
but may be a substantive. This substantive is usually
employed in its intensive capacity, though this is not
necessarily the case. For example, in the proposition
' His name is John ' the predicate is not intended to

convey the idea of any attributes at all.   What is meant to be asserted is that the name of the person in question is that particular name, John, and not Zacharias or Abinadab or any other name that might be given him.

Let it be noticed that when a proposition is read in extension the predicate contains the subject, whereas when it is read in intension the subject contains the predicate.   Viewed externally, or in reference to the breadth of the notion, 'ruminant' contains 'cow'; viewed internally, or in reference to their depth, 'cow' contains 'ruminant.'

Exclusive and Exceptive Propositions.

§ 54. An Exclusive proposition denies the predicate of all but the subject, e. g.

> None but the good are happy.

It is generally an E proposition with a negative term for its subject—

> No not-good men are happy.
>
> No not-S is P.

An Exceptive proposition affirms the predicate of the whole subject with the exception of a certain part, e. g.

> All is lost save honour.
>
> All but two of my pawns are taken.

It is an A proposition with a negative term for its subject.

> All not-honour is lost.
>
> All not-S is P.

An exclusive proposition may always be turned into an exceptive by changing the quality of its predicate.

No one but the sage is sane = Every one but the sage
is mad.

No not-S is P   = All not-S is not-P.

One of these propositions we shall find later to be what
is called the *obverse* of the other.

The exclusive proposition is sometimes introduced by
the word 'alone,' e. g. 'The good alone are happy.'

The word 'except' may occur either in an exceptive
or in an exclusive proposition.

All the Catholic Epistles except Second Peter are
genuine.   Exceptive.

No one except yourself would have done this.
Exclusive.

It is a nice question what is the exact meaning con-
veyed by the exclusive proposition.   When we say 'None
but the good are happy,' are we asserting that good men
are happy?   We certainly seem to do this, especially if we
throw the proposition into the form 'The good alone are
happy.'   It then appears to convey two statements at once—

(1) The good are happy.

(2) No one else is.

The first statement however must not be taken for more
than it is worth.   If quantified, it only amounts to 'Some
good men are happy.'   This proposition follows as a
formal consequence from 'All happy men are good,'
which may be deduced from the original proposition;
whereas 'All good men are happy' does not follow from
it.   But what if the Pessimist be right, and no one at all
is or can be happy?   In that case the first statement

is not true, but we have none the less asserted it. If we use terms which have nothing real to correspond to them, our statements will naturally turn out to be unreal.

The exceptive proposition in like manner seems to convey two distinct statements. E. g. ' All the judges, except two, condemned the prisoner ' is equivalent to—

(1) Two of the judges did not condemn the prisoner.
(2) All the rest did.

The exclusive proposition, though generally E, is not confined to that form. ' All men alone are rational animals ' is an exclusive proposition ; so is the form of I which implies O—Some S only is P. This last is the I proposition of common conversation. When, for instance, we say ' Some of the gates into the park are closed at nightfall,' we are understood to mean ' Some are left open.'

Tautologous or Identical Propositions.

§ 55. A Tautologous or Identical proposition affirms the subject of itself—

Every A is A.
I am I.

Here we have the proposition at the minimum of meaning.

Sometimes a proposition of this form has more meaning than appears at first sight, e. g. ' A man 's a man,' which signifies that a man is not to be judged by the accidents of birth or fortune ; ' What I have written, I have written,' in which the implication is that the writing will not be altered.

CHAPTER IV.

Of the Distribution of Terms.

§ 56. THE treatment of this subject falls under the second part of logic, because distribution is not an attribute of terms in themselves, but as used in a proposition.

A term is said to be distributed when it is known to be used in its whole extent, i.e. with reference to all the things of which it is a name.

When it is used only in part of its extent, or is not known to be used in the whole, it is called undistributed.

§ 57. About the subject of a proposition there can be no difficulty.

If the proposition is universal, the subject is distributed.

If the proposition is particular, the subject is undistributed.

If the proposition is indefinite, the subject has to be taken as undistributed, unless the matter determines otherwise.

§ 58. The predicate however cannot be disposed of quite so easily.

If the proposition is affirmative, the predicate has always to be taken as undistributed, on the same principle as the subject of an indefinite proposition (§ 49). For there is nothing to determine its quantity. 'All sheep are ruminants' is true, if sheep chew the cud. Its truth is

not affected by the question whether sheep are some
or all of the animals that possess this attribute. Generally
then we may say that 'All S is P' may mean, when
taken in extension—

(1) All S is some P.

(2) All S is all P.

As we do not know which it means, we are bound to
take it at its lower value.

On the other hand the predicate of a negative pro-
position is always distributed. 'No S is P,' when taken
in extension, can only mean that S is excluded from
the whole of P. For suppose it were only excluded
from part, then there would be some P left which is S,
and consequently some S which is P, which contradicts
the original assertion. 'No horses chew the cud' would
be upset, if it were admitted that a single animal which
chews the cud is a horse. Hence the proposition must
be interpreted to mean 'No horses are *any* cud-chewers.'

§ 59. The difference between the use of a predicate
in an affirmative and in a negative proposition may be
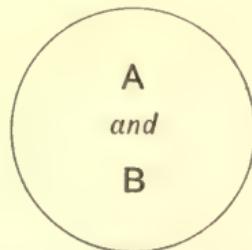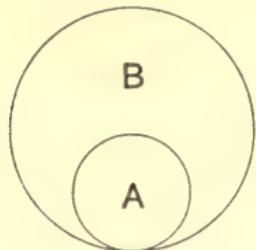
illustrated to the eye as follows. To say 'All A is B'
may mean either that A is included in B or that A and B
are exactly co-extensive.

As we cannot be sure which of these two relations of A to B is meant, the predicate B has to be reckoned undistributed, since a term is held to be distributed only when we know that it is used in its whole extent.

To say 'No A is B' however is to say that A falls wholly outside of B, which involves the consequence that B falls wholly outside of A.
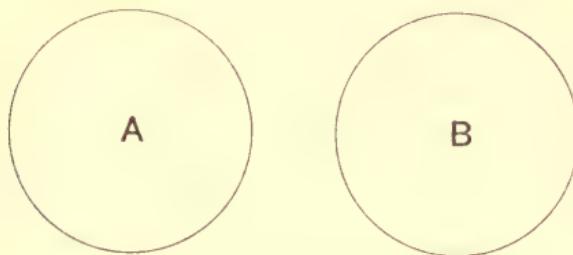
Let us now apply the same mode of illustration to the particular forms of proposition.

There are, from the point of view of extension, two things which may be meant when we say 'Some A is B'—
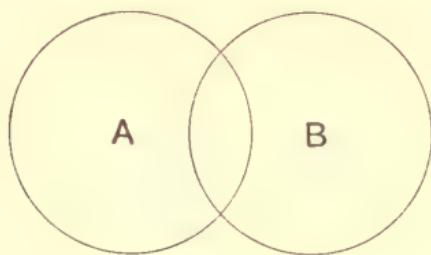
(1) That A and B are two classes which overlap one another, that is to say, have some members in common, e. g. 'Some cats are black.'

(2) That B is wholly contained in A, which is an
inverted way of saying that all B is A, e. g. 'Some
animals are men.'
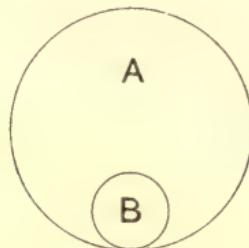
Since we cannot be sure which of these two is meant,
the predicate is again reckoned undistributed.

The above are all that I ever *means*, but it is not
incompatible with the two relations of terms exhibited in
the diagrams already given for A.

All that O ever *means* is one of these two things—
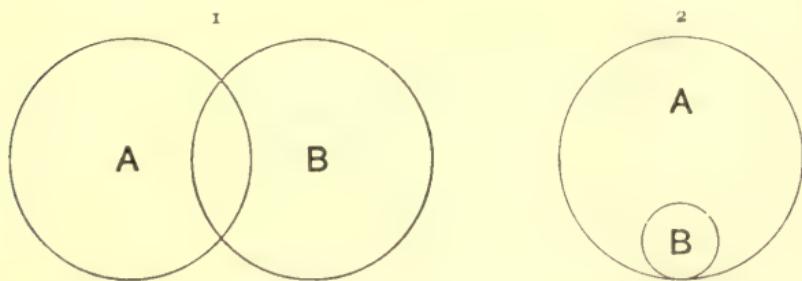
but it is not incompatible with the relation of total ex-
clusion represented in the diagram for E.

§ 60. From the above considerations we elicit the
following

Four Rules for the Distribution of Terms.

1. All universal propositions distribute their subject.
2. No particular propositions distribute their subject.

3. All negative propositions distribute their predicate.

4. No affirmative propositions distribute their predicate.

The distribution or non-distribution of the subject, it will be observed, depends upon the quantity, that of the predicate upon the quality, of the proposition.

The above four rules are the basis upon which the rest of the edifice is built. They therefore merit particular attention. When applied to the four forms of proposition, they lead to the following results—

A distributes its subject only,

E distributes both subject and predicate,

I distributes neither subject nor predicate,

O distributes its predicate only.

These results have been embodied in the following help to the memory—

As Eb In Op:

but it is on the four rules themselves, rather than on the application of them, that it is important to fix the attention.

CHAPTER V.

Of the Quantification of the Predicate.

§ 61. ATTEMPTS have been made in recent times to
build logic on another foundation. Sir William Hamilton
maintained that, although the predicate is not quantified
in language, it must always be quantified in thought. If
this were so, we should require eight, instead of four,
forms of proposition. For hitherto we have taken account
only of the quantity of the subject in determining the
quantity of a proposition, but on this view we should have to
take account equally of the quantity of the predicate. The
eight propositions which result, together with the symbols
which have been devised for them, are as follows—

$$A \begin{cases} \text{1. All S is all P.} & (\text{U}). \\ \text{2. All S is some P.} & (\text{A}). \end{cases}$$

$$E \begin{cases} \text{3. No S is any P.} & (\text{E}). \\ \text{4. No S is some P.} & (\eta). \end{cases}$$

$$I \begin{cases} \text{5. Some S is all P.} & (\Upsilon). \\ \text{6. Some S is some P.} & (\text{I}). \end{cases}$$

$$O \begin{cases} \text{7. Some S is not any P.} & (\text{O}). \\ \text{8. Some S is not some P.} & (\omega). \end{cases}$$

It is evident that it is the second of the above pro-
positions which represents the original A, in accordance
with the rule that 'No affirmative propositions distribute
their predicate' (§ 60).

The third represents the original E, in accordance with

the rule that 'All negative propositions distribute their predicate.'

The sixth represents the original I, in accordance with the rule that 'No affirmative propositions distribute their predicate.'

The seventh represents the original O, in accordance with the rule that 'All negative propositions distribute their predicate.'

Four new symbols are required, if the quantity of the predicate as well as that of the subject be taken into account in the classification of propositions. These have been supplied, somewhat fancifully, as follows—

The first, 'All S is all P,' which distributes both subject and predicate, has been called υ, to mark its extreme universality.

The fourth, 'No S is some P,' is contained in E, and has therefore been denoted by the symbol η, to show its connexion with E.

The fifth, 'Some S is all P,' is the exact converse of the second, 'All S is some P,' and has therefore been denoted by the symbol Υ, which resembles an inverted A.

The eighth is contained in O, as part in whole, and has therefore had assigned to it the symbol ω.

§ 62. The attempt to quantify the predicate leads to some curious results. Let us take, for instance, the υ proposition. Either the sign of quantity 'all' must be understood as forming part of the predicate or not. If it is not, then the υ proposition 'All S is all P' seems to contain within itself, not one proposition, but two, namely,

'All S is P' and 'All P is S.' But if on the other hand 'all' is understood to form part of the predicate, then υ is not really a general but a singular proposition. When we say, 'All men are rational animals,' we have a true general proposition, because the predicate applies to the subject distributively, and not collectively. What we mean is that 'rational animal' may be affirmed of every individual in the class, man. But when we say 'All men are all rational animals,' the predicate no longer applies to the subject distributively, but only collectively. For it is obvious that 'all rational animals' cannot be affirmed of every individual in the class, man. What the proposition means is that the class, man, is co-extensive with the class, rational animal. The same meaning may be expressed intensively by saying that the one class has the attribute of co-extension with the other.

Under the head of υ come all propositions in which both subject and predicate are singular terms, e. g. 'Homer was the author of the Iliad,' 'Virtue is the way to happiness.'

The proposition η conveys very little information to the mind. 'No S is some P' is compatible with the A proposition in the same matter. 'No men are some animals' may be true, while at the same time it is true that 'All men are animals.' No men, for instance, are the particular animals known as kangaroos.

The ω proposition conveys still less information than the η. For ω is compatible, not only with A, but with υ. Even though 'All men are all rational animals,' it is still

true that 'Some men are not some rational animals': for no
given human being is the same rational animal as any other.

Nay, even when the υ is an identical proposition, ω will
still hold in the same matter. 'All rational animals are
all rational animals:' but, for all that, 'Some rational
animals are not some others.' This last form of pro-
position therefore is almost wholly devoid of meaning.

§ 63. That the eight forms of proposition are too many
may be argued also from the fact that they outrun the possi-
ble relations of terms to one another in point of extension[1],
as may be gathered from the following scheme:
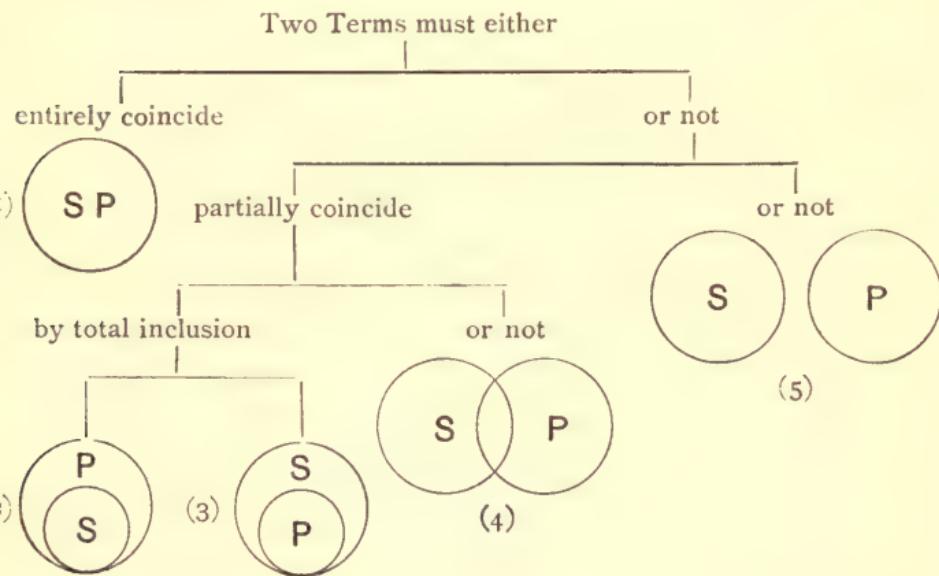
The four ordinary propositions, A, E, I, O, cover the
whole ground between them, A corresponding to (1)
and (2), I and O to both (3) and (4), and E to (5).

[1] On this point I am indebted to Mr. Keynes, *Formal Logic*,
1st ed. § 95.

It is certainly useful on occasions to distinguish υ from A, possibly also Υ from I : but η and ω may fairly be regarded as of no practical value. η is satisfied by (2), (4), and (5), while ω covers the ground of all.

§ 64. The chief advantage claimed for the quantification of the predicate is that it reduces every affirmative proposition to an exact equation between its subject and predicate. As a consequence every proposition would admit of simple conversion, that is to say, of having the subject and predicate transposed without any further change in the proposition. The forms also of Reduction (a term which will be explained later on) would be simplified ; and generally the introduction of the quantified predicate into logic might be attended with certain mechanical advantages. The object of the logician however is not to invent an ingenious system, but to arrive at a true analysis of thought. Now, if it be admitted that in the ordinary form of proposition the subject is used in extension and the predicate in intension, the ground for the doctrine is at once cut away. For, if the predicate be not used in its extensive capacity at all, we plainly cannot be called upon to determine whether it is used in its whole extent or not.

CHAPTER VI.

Of the Heads of Predicables.

§ 65. A PREDICATE is that which is said of a subject.

A predicable is anything which can be said of a subject.

The Heads of Predicables are the different kinds of things which can be said of a subject.

The Heads of Predicables then are meant as a classification of the different relations in which the predicate can stand to the subject in a proposition. Hence the treatment of them falls under the second part of logic. These relations are reckoned as five—

> Genus,
> Species,
> Difference,
> Property,
> Accident.

§ 66. We will begin by defining these terms as they are used at the present day, and will afterwards inquire into their original meaning.

Genus is a larger class viewed in relation to some smaller class contained under it.

> (genus)
> All men are animals.

Species is a smaller class viewed in relation to some larger class under which it is contained.

> (species)
> Some animals are men.

Under the first two heads, it will be observed, we have a predication of classes, expressed by a noun substantive; under the remaining three we have a predication of attributes, expressed by a noun adjective.

Difference (*differentia*) is the attribute or attributes which mark off a species from a genus.

<div align="center">
(difference)
Men are rational.
</div>

By this attribute man is supposed to be distinguished from all other animals, which are called 'brutes.'

<div align="center">
N.B. Genus + Difference = Definition.
(difference) (genus)
Men are rational animals.
</div>

Property is an attribute which is not contained in the definition of a term, but which follows from it.

A generic property is one which follows from the genus.

<div align="center">
(generic property)
Men have appetites.
</div>

It is as animals that men have appetites.

A specific property is one which follows from the difference.

<div align="center">
(specific property)
Men are capable of studying logic.
</div>

It is in virtue of their rationality that men are capable of studying logic. A cow has no aptitude in this direction.

Accident is an attribute which is neither contained in the definition of a term nor follows from it.

An inseparable accident is one which belongs to all the members of a class.

(inseparable accident)
Animals which chew the cud divide the hoof.

A separable accident is one which belongs only to some members of a class.

(separable accident)
Some animals have four legs.

Blackness is a separable accident of man, an inseparable accident of coals.

The distinction between inseparable and separable accidents is also applied to an individual.

An inseparable accident of an individual is one which belongs to it at all times, e.g. it is an inseparable accident of a particular person that he is a native of India.

A separable accident of an individual is one which belongs to it only at a certain time or times, e.g. it is a separable accident of a particular person that he is visiting India.

Again it is an inseparable accident of this pen that it is made of steel; it is a separable accident that it is being employed in writing.

§ 67. The attributes which belong to anything may be distinguished broadly under the two heads of essential and non-essential, or accidental. The essential attributes are those which are contained in, or which flow from, the definition. Now it may be questioned whether there can, in the nature of things, be such a thing as an inseparable accident. For if an attribute were found to belong invariably to all the members of a class, we should

suspect that there was some causal connexion between it and the attributes which constitute the definition, that is, we should suspect the attribute in question to be essential and not accidental.   Nevertheless the term 'inseparable accident' may be retained as a cloak for our ignorance, whenever it is found that an attribute does, as a matter of fact, belong to all the members of a class, without there being any apparent reason why it should do so.   It has been observed that quadrupeds which have horns chew the cud.   As no one can adduce any reason why animals that have horns should chew the cud any more than animals which have not, we may call the fact of chewing the cud an inseparable accident of horned animals.   If there really is no reason why, then it *is* an inseparable accident, and belongs to the domain of uniformities which are not results of causation.

§ 68. It must be noticed that we have not really defined the term 'accident,' not having stated what it is, but only what it is not.   It has in fact been reserved as a residual head to cover any attribute which is neither a difference nor a property.

§ 69. In dealing with the above classification it is important to bear in mind that it assumes an acquaintance with the 'essences' of things, that is, in modern language, it assumes a knowledge of definitions.   Take this away and the whole thing becomes unintelligible.   When, for instance, we say 'Man is an animal,' 'Man is rational,' 'Man is progressive,' there is nothing in the nature of

these statements themselves, except indeed the substantival form of the first, to tell us that the predicate is genus, difference, or property respectively. It is only by a tacit reference to the accepted definition of man that this becomes evident to us. Similarly we cannot know from merely hearing the word 'triangle' that the fact of a triangle having three sides is its difference, and the fact of its having three angles a property. There is a reason for defining it in one way rather than another : but, if we are unacquainted with it, there is nothing to show us that the three-corneredness may not be the difference and the three-sidedness the property. For these two attributes are so connected together that, whichever is postulated, the other will necessarily follow.

§ 70. If the above five terms were presented as an exhaustive classification of the possible relations in which the predicate can stand to the subject in a proposition, there are certain obvious criticisms that might be made.

(1) No notice is taken of the case in which the predicate is a singular term.

In such a proposition as 'This man is John,' we have neither a predication of class nor of attribute, but merely the identification of one term with another as applying to the same object.

(2) It is defective even as regards general predication. For

(*a*) there is no room for the case in which the

predicate is a mere synonym for the subject, e.g.

A giraffe is a camelopard;

(*b*) it takes no account of those forms of predication in which class and attribute are combined.

Under which of the five heads would the predicates in the following propositions fall?

Man is a rational animal.
Man is a featherless biped.

(3) It altogether omits particular propositions.

§ 71. But, if we were to confine predication to universal propositions and to common terms, and were to grant that a predicate must always take the form of class or attribute and never of both, the classification would then admit of defence by logic. For when the predicate is a class, the term predicated is called a Genus, if the subject itself be a class, or a Species, if it be an individual[1]. When, on the other hand, the predicate is an attribute, it may be either the very attribute which distinguishes the subject from other members of the same class, in which case it is called the Difference, or it may be some attribute connected with the definition, i.e. Property, or not connected with it, i.e. Accident.

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[1] Here it will be observed that we are using 'species' in a different sense from that in which it was defined above. In the language of the Schoolmen it is now *species praedicabilis*, whereas before it was *species subjicibilis*.

These results may be exhibited in the following
scheme —
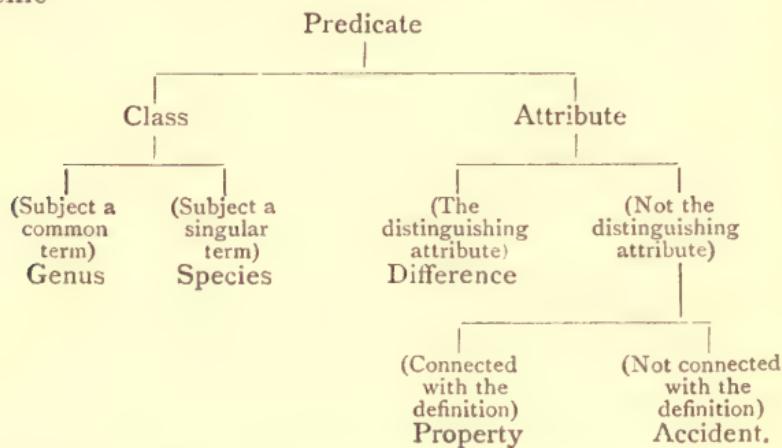
Such criticism however and such defence would be
equally beside the mark, the fact being that the five terms
have been applied to a purpose for which they were not
originally intended.

The Five Words of Porphyry.

§ 72. The Heads of Predicables, as usually given, are
not taken from Aristotle, but from the *Introduction* of
Porphyry, a Neo-Platonic philosopher, who wrote some
six centuries later. But Porphyry's own object in writing
this short work was not to give a list of Heads of
Predicables, but merely to supply an easy introduction
to Aristotle's treatise on the Categories and to logic
generally by explaining the five words—genus, species,
difference, property, accident. These are five general
predicates, which he begins by distinguishing from
singular predicates, like ' Socrates,' ' this man,' ' this
thing.' Had his object then been to classify the relations

in which a predicate can stand to a subject in a proposition, he would certainly have given us six and not five heads. But in point of fact he had no such object.

§ 73. On the metaphysical questions connected with what was afterwards known as Realism and Conceptualism Porphyry professes not to enter: but his assumption throughout is that classes exist in nature with real differences to mark them, which in no way depend upon our minds.

§ 74. The 'tree of Porphyry' is a device added by later writers[1], but in the text of the treatise the category of substance is arranged as follows—

<div align="center">

Substance
|
Body
|
Living body
|
Animal
|
Rational animal
|
Man
|

</div>

| | | |
| Socrates | Plato | &c. |
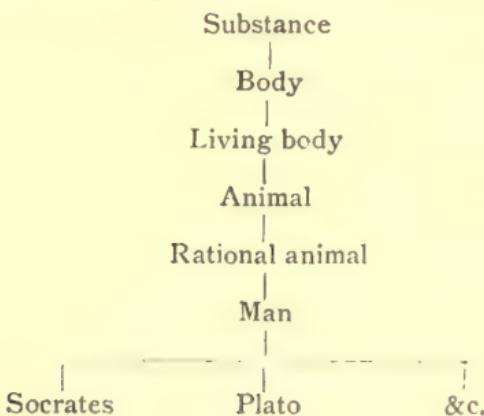
Here Substance is the *summum genus* or *genus generalissimum* (γενικώτατον γένος), which is always genus and never species, as having no class above it[2]. Man is an

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[1] For the most graphic picture of the 'tree' see Father Clarke's *Logic*, p. 181.

[2] We might suppose that 'thing' or 'being' could be predicated of 'substance,' but Porphyry, following Aristotle, regards each of the ten categories as a distinct *summum genus*. He will not allow that 'being' is predicable of them all in the same sense.

infima species (εἰδικώτατον εἶδος), which is always species and never genus, as having no real classes below it. The intermediate terms are Subaltern Species and Genera (ὑπάλληλα εἴδη καὶ γένη), being species in relation to the classes above them, genera in relation to those below them.

Hence 'species' is used in two senses—

(1) For any class in reference to a higher class under which it falls.

(2) For the lowest class in reference to individuals.

Genus (γένος) and Species (εἶδος) are so essentially relative, that Porphyry does not attempt to define one apart from the other. His definition of genus is 'that which is predicated in the category of substance of several things different in kind (=species),' and his definition of species is 'that which is arranged under the genus, and of which the genus is predicated in the category of substance.' But *infima species* is defined as 'that which is predicated in the category of substance of several things numerically different.'

This last definition is most conformable to such natural kinds as metals and other homogeneous substances, in which the members of the same class differ only numerically. A heterogeneous substance, like man, presents so many points of individual difference, that here we feel no bar to further subdivision.

§ 75. Neither of course did Porphyry. But he distinguishes between 'specific' and ordinary differences. It is only the former which mark off real kinds. Add

rationality to animality, and you get man, a being different in species from a horse : but add blackness or a snub nose to man, and you get a difference indeed, but not such as to mark a new species.

Since any attribute may form a difference between things in the loose sense of the term, Difference (διαφορά) is used by Porphyry for attribute generally. In this sense differences are divided into Separable, such as motion and rest, health and disease, and Inseparable, such as hook-nosed and snub-nosed, rational and irrational. Inseparable differences are subdivided into Accidental and Essential, hook-nosed and snub-nosed being instances of the former, rational and irrational of the latter.

Essential differences are those which are contained in the definition of a thing, or which follow from it. Thus rational and mortal and the being receptive of knowledge are essential differences of man. The first two are contained in the definition, the third follows from it. For Porphyry's definition of man is 'a rational, mortal animal.' It follows from his reason that he is receptive of knowledge.

Further we are told that essential differences do not admit of degree, whereas accidental do. A being may be more or less black, or more or less snub-nosed, but not more or less rational. This is a hard saying, but, unless it be accepted, what is to become of the species fixed by nature ? The great law of continuity is fatal to this ancient realism, which has indeed all along been a protest against the flux of matter. If nature's species run into

one another, it would seem realism must be interpreted as a theory of types, by approximation to or recedence from which the place of a sensible thing in the hierarchy of nature has to be determined.

Essential differences are subdivided into Constitutive and Divisive. On the whole then we get this scheme—

```
                          Difference
                              |
        _____
        |                                               |
   Separable                                        Inseparable
                                                         |
                                  _____
                                  |                                     |
                             Accidental                            Essential
                                                                         |
                                               _____
                                               |                                        |
                                          Constitutive                              Divisive.
```
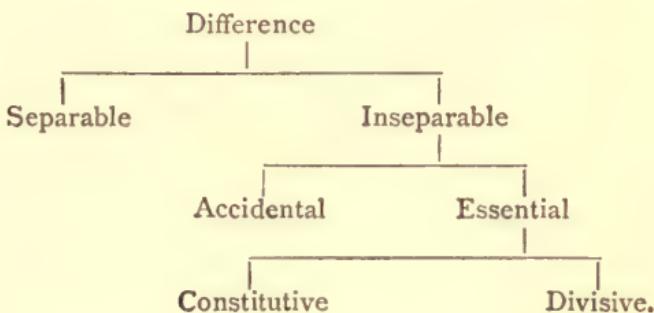
To illustrate the distinction between constitutive and divisive differences, let us take the case of 'animal.' An animal is a living, sensitive substance; it is also rational or irrational, mortal or immortal. Here 'living' and 'sensitive' are differences which constitute 'animal,' whereas 'rational' and 'irrational,' 'mortal' and 'immortal' are differences which divide it. But the divisive differences of a genus become constitutive of a species. Thus, when added to the genus animal,

$$\begin{aligned}
\text{rational} &+ \text{mortal} &= \text{man} \\
\text{rational} &+ \text{immortal} &= \text{god} \\
\text{irrational} &+ \text{mortal} &= \text{brute} \\
\text{irrational} &+ \text{immortal} &= ?
\end{aligned}$$

Porphyry refrains from adding a fourth species to contain such individuals as Cerberus and Pegasus. It is curious

to note that towards the end of the treatise he substitutes 'angel[1]' for 'god,' as falling along with man under the difference 'rational.'

Essential differences, whether constitutive or divisive, are alike specific. This means that there are real attributes, whereby classes are marked off from one another, and that it is our business to find these, instead of dividing as we please.

A Specific or Essential Difference is defined in two ways, as

  (1) that whereby the species exceeds the genus, e.g. man exceeds animal by rational + mortal;
  (2) that which is predicated in the category of quality of several things different in kind, e.g. rational is so predicated both of man and god.

§ 76. Both genus and difference form part of the essence (οὐσία) of a thing, but in different ways, the genus being analogous to the matter and the difference to the form. As a sculptor carves a statue out of a block of marble by imposing form upon it, so from the rough matter of 'animal' nature carves out the species 'man' by adding thereto 'rational' and 'mortal.' Hence genus is called the Material and difference the Formal part of the essence.

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[1] Porphyry (A. D. 263), the great opponent of Christianity, is himself suspected of Christian antecedents. Simplicius (A. D. 500), another Pagan writer, speaks in the same way of 'angelic' or divine virtues (*Comment.* in *Epict. Ench.* cap. viii). It does not therefore seem necessary to suppose that the text has been tampered with.

§ 77. The genus potentially contains all the divisive differences. If it did not potentially contain them, whence could the species get them? If it actually contained them, the law of contradiction would be violated.

§ 78. Of Property (ἴδιον) there are four kinds recognised by Porphyry—

(1) that which belongs only to one species, though not to all the members of it,

e.g. medicine or geometry is a property in this sense of man;

(2) that which belongs to a whole species, though not to that only,

e.g. being a biped is a property in this sense of man;

(3) that which belongs to a whole species, and to that only, though not at all times,

e.g. the growing grey in old age is a property in this sense of man;

(4) that which belongs to a whole species, and to that only, and at all times,

e.g. a capacity for laughter is a property in this sense of man.

This last is the strict sense of the word ' property.' Let us call it for distinction *proprium*, or ' peculiar property.'

§ 79. Accident (συμβεβηκός) is defined as ' that which may be there or not without destruction to the subject.'

Accidents are divided into Separable and Inseparable. To be asleep is a separable accident of an animal. Blackness is an inseparable accident of a crow or of an

Ethiopian : still it is a mere accident, for we can conceive a crow to be white or an Ethiopian to change his skin. Accident is also defined negatively as 'that which is neither genus nor difference nor species nor property, but which always exists in the subject.'

§ 80. This account of Porphyry's teaching on the subject of the five words may fitly be concluded by the neat summary of it which was made by the Scholastic Philosophy [1]—that every common term, in relation to individuals, signifies either their whole essence, or part of their essence, or something joined to their essence.

If it signifies their whole essence, it is Species.

If it signifies the material part of their essence, it is Genus.

If it signifies the formal part of their essence, it is Difference.

If it signifies something necessarily joined to their essence, it is Property.

If it signifies something contingently joined to their essence, it is Accident.

Nothing could be more satisfactory than this, if we only knew what the essence of a thing was. If we agree to call that the essence of a thing which is contained in its accepted definition, we may still use the classification.

§ 81. In spite of its name the subject of Heads of Predicables was treated by the Schoolmen under the first part of Logic, as dealing with the relations of terms to one another, in themselves rather than in the proposition.

[1] See Father Clarke's *Logic*, p. 175.

Aristotle's Heads of Predicables.

§ 82. Aristotle himself really did aim at the object which we have considered to be proper to the Heads of Predicables, namely, to classify the relations in which a predicate can stand to a subject. Accordingly there is room in his division for the mixed forms of predication before referred to (§ 70). His list [1] contains four heads, namely,

> Genus (γένος),
> Definition (ὅρος),
> *Proprium* (ἴδιον),
> Accident (συμβεβηκός).

§ 83. At first he gives only three, saying that every proposition in a syllogism, whether premiss or conclusion, signifies either genus, property, or accident. Property is here a vague term covering any predicate which is convertible with the subject. By splitting this up into definition and *proprium* [2], he reaches the fourfold list

> Genus
> Property $\left\{ \begin{array}{l} \text{Definition} \\ \textit{Proprium} \end{array} \right.$
> Accident.

§ 84. But what, it may be asked, has become of differ-

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[1] *Top.* I. 4, §§ 1, 2; 5, § 1.

[2] *Proprium* is used here for convenience. With Aristotle the same word ἴδιον is used in the wider and in the narrower sense.

ence, which is surely a legitimate predicable? The answer is that it is absorbed into genus. Whether we say 'Man is an animal' or 'Man is rational' we are equally referring him to something wider than himself.

Species too is absent from the list. But has it a right to be there? When predicated of an individual, it is indistinguishable from genus, except on the assumption of known *infimae* species; when predicated of a class, it is an accident, e.g. 'Some animals are horses.'

§ 85. Aristotle's classification may easily be seen to be exhaustive. For every predicate must either be co-extensive with its subject or not, i.e. predicable of the same things. And if the two terms coincide in extension, the predicate must either coincide also in intension with the subject or not.

A predicate which coincides both in extension and intension with its subject is of the nature of a Definition. One which coincides in extension without coinciding in intension, that is, which applies to the same things without expressing the whole meaning of the subject, is what is known as a *Proprium* or Peculiar Property.

If, on the other hand, the two terms are not co-extensive, the predicate must either partially coincide in intension with the subject or not[1]. This is equivalent

[1] The case could not arise of a predicate which was entirely coincident in intension with a subject with which it was not co-extensive. For, if the extension of the predicate were greater than that of the subject, its intension would be less, and if less, greater, in accordance with the law of inverse variation of the two quantities (§ 37).

to saying that it must either state part of the definition of the subject or not. Now the definition is made up of genus and difference, either of which may form the predicate : but as the two are indistinguishable in relation to a single subject, they are lumped together for the present purpose under the one head, Genus. When the predicate, not being co-extensive, is not even partially co-intensive with its subject, it is called an Accident.

§ 86. In the work in which this classification occurs Aristotle expressly disclaims scientific precision of treatment. The want of exactness however is not discernible in the division itself, but in his not having suited his definitions to the members of it. He prefers to define the four terms in what was already their accepted sense, and to aggregate doubtful cases round whichever of them they were most like.

§ 87. Genus is defined in the same way as by Porphyry, as 'that which is predicated in the category of substance of several things different in kind.'

Under this head difference is thrown, although it does not comply with the definition, since it is predicated in the category of quality. Generic properties (§ 131) may also be referred to the same head. It is a generic property of man that he possesses irrational impulses, since this follows from his nature as an animal (genus).

§ 88. Definition is defined as 'a phrase which signifies the nature of a thing[1].'

Under this head must be included any predicate which

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[1] *Top.* I. 5, § 1 λόγος ὁ τὸ τί ἦν εἶναι σημα'νων.

is a mere synonym of the subject, e. g. 'A mantle is a cloak,' 'A Hebrew is a Jew,' 'The skipper is the captain,' 'Alexander is Paris.' In such propositions the predicate coincides in extension with the subject, and may be considered to coincide in intension, where the intension of both subject and predicate is at zero, as in the case of proper names.

§ 89. *Proprium* is defined as 'that which does not indeed make plain the nature of a thing, but which belongs to that thing only, and can be predicated instead of it,'

> e.g. 'able to learn grammar' can be said only of man and can be put in place of 'man.'

Designations (§ 31) and descriptions (§ 119) will fall under this head, e.g. 'Mr. Balfour is the present Prime Minister of England,' 'Man is a mammal with hands and without a hairy skin.' Here, while the terms are coincident in extension, they are far from being so in intension.

§ 90. Accident is defined—

> (1) negatively, as 'that which is neither definition nor *proprium* nor genus, but which belongs to the thing;'
> (2) positively, as 'that which may belong or not belong to one and the same individual.'

The second of these definitions is declared to be the better, since it is complete in itself, whereas the first postulates a knowledge of the other heads of predicables.

§ 91. These results may be exhibited in the following scheme—
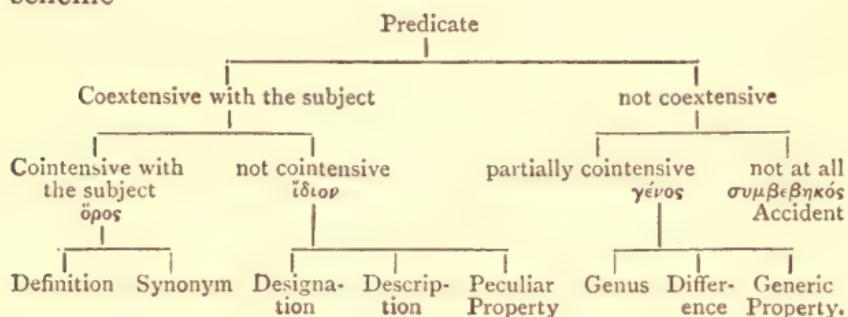
Thus Aristotle's four heads of predicables may be split up, if we please, into nine—

       1. Definition   ⎫
       2. Synonym   ⎬ ὅρος.

       3. Designation
       4. Description   ⎬ ἴδιον.
       5. Peculiar Property

       6. Genus
       7. Difference   ⎬ γένος.
       8. Generic Property

       9. Accident—συμβεβηκός.

CHAPTER VII.

Of the Categories.

§ 92. THE Four Heads of Predicables, Aristotle tells us, will always be found in the Ten Categories. These last are certain *summa genera* which contain all possible predicates [1]. Hence they were known among the Schoolmen as the Ten Predicaments.

§ 93. While the heads of predicables consider a predicate in relation to its subject, the categories look at it in itself.

§ 94. The categories in a more or less complete form occur everywhere in the Aristotelian writings, but they have also been thought worthy of a special treatise, from which we extract a passage, as the simplest way of bringing the subject before the reader—

'Every word used by itself signifies either substance or quantity or quality or relation or place or time or posture or having or doing or suffering. As an example of substance we may take "man," "horse;" of quantity "two cubits long," "three cubits long;" of quality "white," "grammatical;" of relation "double," "half," "greater;" of place "in the Lyceum," "in the Agora;" of time "yesterday," "last year;" of posture "lying," "sitting;" of state "shod," "armed;" of doing "cutting," "burning;" of suffering "being cut," "being burned."'

[1] τὰ γένη τῶν κατηγοριῶν, *An. Post.* I. 22, § 7; *Top.* I. 9, § 1.

§ 95. The Greek names for the ten categories with their Latin equivalents are as follows—

οὐσία	substantia	substance
ποσόν	quantitas	quantity
ποιόν	qualitas	quality
πρός τι	relatio	relation
ποῦ	ubi	place
πότε	quando	time
κεῖσθαι	situs	posture
ἔχειν	habitus	having
ποιεῖν	actio	doing
πάσχειν	passio	suffering.

§ 96. Mill (*Logic* I. 3, § 1) has brought some damaging criticisms to bear upon these categories when given (as they are in the *Metaphysics* IV. 7, § 4) as an enumeration of Existences: but he seems to have overshot the mark in saying that the distinction between *Ubi* and *Situs* is merely verbal. For *Situs* means the 'lie' of a thing, that is to say, its posture, or, as we should call it in the case of a person, its attitude. There is a difference of conception, and not a mere verbal difference, between this and the idea of place. A spinning-top asleep does not change its place, but its *Situs* is changing every instant. Or again, a backgammon die occupies the same place, whichever of its sides be uppermost, but the latter question will make all the difference to the game. It is true that the posture of anything depends upon the place which its parts occupy

relatively to one another, but in reference to the object itself the ποῦ and the κεῖσθαι, its place and its posture, are distinct conceptions.

§ 97. Aristotle gives us no clue to the way in which he came by the categories. There they are, and there they seem to have been from the beginning, in his philosophy. But it is plain that his direct object in forming them was not to work out a classification of things in general. Κατηγορία means 'predication,' and to speak of the 'classes of predication,' or 'the forms of predication' (*Met.* IV. 7, § 4), only gives the term a more extended, but not a different meaning. It might be thought therefore that the problem which Aristotle set himself to solve was this—What different notions can be affirmed or denied of the subject by means of the copula? But to put the matter in this way would be to overlook the great difference between ancient dialectic and modern logic. A syllogism with us means a combination of three propositions. But in Greek dialectic there was no proposition at all in the syllogism. For the conclusion was propounded as a question, in a form which left the matter open to discussion, e.g. 'Is pleasure a good or not?'—and then assent to one side or the other was obtained by putting questions which demanded the answer 'Yes' or 'No,' e.g. 'Is every good praiseworthy?' The conclusion, as thus put forward interrogatively, was technically known as a 'problem,' and each of the questions which determined the answer to it was called a *protasis.*

§ 98. Now the task which Aristotle set himself to accomplish was not to make out a list of all possible objects of thought, but rather this—Given any object of thought, what are the main questions which can be asked about it and the answers that can be returned to them?

To begin with, we may ask what a thing is, and be told that it is a horse or a man. That is the category of Substance. Or we may ask 'How big is it?' and receive the answer 'fourteen hands' or 'six feet high.' That is the category of Quantity. Or we may request information about its nature, and be told that it is spirited or learned. That is the category of Quality. Or we may want to know the relation it bears to something else, and be told that it is like something else or the servant of somebody. That is the category of Relation. Or we may ask 'Where is it?' and receive the answer 'In the market-place.' That is the category of Place. Or, if the thing in question be an event, we may inquire when it took place, and be told 'yesterday' or 'A year ago.' That is the category of Time. Or we may ask how its parts stand with regard to one another, and receive the information that it is upside-down. That is the category of Posture. Or we may be interested in its belongings and may learn that it has shoes on or a ring on its finger. That is the category of Having. Or we may ask what it is engaged in and get the reply that it is eating or teaching. That is the category of Doing. Or, lastly, we may want to know

how it is being acted on by something else, and be told that it is being eaten or being taught. That is the category of Suffering.

§ 99. That this was really the way in which Aristotle arrived at his categories may be gathered from the interrogative forms by which they are designated. Thus the category of Substance is commonly spoken of as the category of 'What is it?' the next as the category of 'How big?' and so on through the list[1]. Moreover, Time could not have appeared as a category except by looking at the possible answers to questions. For the adverb of time is not categorematic. We cannot predicate it of anything, but we may give it as an independent answer[2] to a question couched in a particular form.

§ 100. It did not escape Aristotle[3] that the question τί ἐστι might really receive its answer under any of the categories. For if the thing of which we are speaking be not a substance, but a quantity or quality or relation, then the answer to the question 'What is it?' comes under one of these other categories. Suppose for instance we ask 'What is maternity?' the answer will be, 'It is the relation of a mother to her child.' It is the οὐσία of maternity that it is a relation, the οὐσία of virtue that it is a quality, and so on. But such things have no οὐσία in the strict sense of substantial existence. This is why

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[1] This fact is commonly obscured by the accentuation.

[2] κατὰ μηδεμίαν συμπλοκήν, Cat. 4, § 1.

[3] *Met.* VI. 4, § 12; *Top.* I. 9, § 2.

Aristotle and Porphyry deny that the ten categories can be brought under one head, except equivocally. The vague term 'thing' enables us to express what is common to all with less risk of ambiguity. It is a pity therefore that this word should be so often used interchangeably with 'substance,' instead of being reserved for the absolute *summum genus* : for everything, even the most abstract quiddity or entity, must still be a thing of some kind.

§ 101. The nine categories other than that of Substance may be regarded as a division of Attribute. Quantity and Quality are attributes inherent in a single substance, the one belonging to its matter, the other to its form, and so both fall under 'quality' in the wide sense in which we have used the term (§ 24). The rest are all relations of one kind or another : but the special category of Relation seems to be reserved by Aristotle for things which have a distinct correlative.

§ 102. As Aristotle's problem was essentially concerned with language, it is not surprising that he should have sought to solve it by aid of grammatical distinctions, and that in so doing he should have struck out the parts of speech, so far as they are capable of supplying an answer to a question. The noun substantive led him to the category of Substance, the noun adjective in its various forms to those of Quantity, Quality, and Relation ; adverbs like 'here' and 'now' to those of Place and Time ; the active and passive voices of the verb obviously supplied the categories of Doing and Suffering ; while

Posture and Having seem from his instances to have been suggested by the neuter verb and the middle voice.

§ 103. We pass on now to the two subjects of

Definition and Division,

in which the utility of the second part of logic culminates.

These two processes correspond to the two kinds of quantity possessed by terms.

Definition is the analysis of a term in intension.

Division is the analysis of a term in extension.

Definition imparts clearness to our thoughts by setting before us the meaning of the terms we are using.

Division imparts distinctness to our thoughts by showing us the different kinds of things that are called by a common name.

Neither process is a purely formal one: both have to take account of the matter of thought. Every definition suggests a division; every division supplies a definition of each of its members. Thus when we have defined man as a rational animal, we have suggested a division of animal into rational and irrational; when we have divided animal into rational and irrational, we have supplied definitions of 'man' and 'brute.'

Definition, involving generalisation, is seeing the one in the many; division is seeing the many in the one.

'I am myself enamoured,' says Socrates in the *Phaedrus* (266 B), 'of these divisions and generalisations, in order that I may be able to speak and think; and if I deem that another is able to see a One and a Many in nature,

him I follow after, in his footsteps, as though he were a god.'

The treatment of the two processes falls under the second part of logic, because it involves an acquaintance with the Heads of Predicables.

CHAPTER VIII.

Of Definition.

§ 104. DEFINITION is of things through names. Hence we speak both of defining things and of defining terms.

To define a thing is to fix upon its most essential attributes.

To define a term is to state the most essential part of its intension.

§ 105. Let it be understood that by 'intension' we mean all the attributes in any way implied or suggested by a term, whether they are essential to the thing of which it is a name or not. A capacity for laughter is part of the intension of the term 'man,' though not essential to a rational animal.

§ 106. By the essential attributes of a thing we mean those without which it would not be what it is. Man would not be man, if he were either not an animal or not rational. He would still be man, if it never occurred to him to laugh; much more so, if his skin were not white. But a disembodied spirit must be called by some other name than 'man;' and a being from whom reason were wholly absent, though in human form, would no more be entitled to be called 'man' than a statue.

§ 107. Among the essential attributes of a thing some are primary and others secondary. Any attribute which can be shown to follow from another is plainly secondary

to that other. Thus a capacity for progress is secondary to rationality among the attributes of man. He is progressive because he is rational, not rational because he is progressive.

The essential and primary attributes of a thing are what constitute the meaning of its name as fixed by definition.

§ 108. *Rules for Definition.*

I. MATERIAL.

The attributes selected must be—

      (1) truly predicable of the thing,

      (2) important,

      (3) fundamental.

    Violations. (1) 'The sun is the brightest of the heavenly bodies that go round the earth.'

This is given by Plato (*Theaet.* 208 D) as a model of what a definition ought to be: but we have changed all that, and no one would now accept the definition, because it is not true.

    (2) 'Man is a featherless biped.'

The story runs that, when Plato had defined man as 'a featherless biped,' Diogenes plucked a cock and introduced it into his school as 'Plato's man.' Thereupon Plato added the further difference 'with broad nails' (Diog. Laert. VI, § 40).

(3) 'A triangle is a figure with three angles.'

In this case the attributes selected are both truly predicable and important, and yet the definition is not a good one. Why? Because the angles depend upon the sides rather than the sides upon the angles. Generally, if attribute B is found to follow from A, we should define by A and not by B.

II. Formal.

(1) A definition must be convertible with the term defined, i. e. it must apply to precisely the same things, neither more nor less.

> Violations. Caviare is a kind of food.
>
> A triangle is a figure with three equal sides.

(2) A definition must be clearer than the term defined.

> Violation. A net is a reticulated fabric, decussated at regular intervals.

The violation of this rule is called *ignotum per ignotius* or *per aeque ignotum.*

(3) A definition must contain the fewest attributes that suffice to distinguish the thing defined.

> Violation. A triangle is a figure with three sides and three angles.

(4) A definition must not be metaphorical.

> Violation. Bread is the staff of life.

(5) A definition must not contain the name defined either directly or by implication.

> Violation. Virtue is acting virtuously.

The violation of this rule is called *circulus in definiendo,* or defining in a circle.

(6) A definition must not be negative, if it can be affirmative.

> Violation. A Turanian language is one which is neither Aryan nor Semitic.

Briefly, a definition

should be convertible with the subject, clear, terse, and should not be metaphorical, tautologous, or negative. The first of these formal rules is the best test that can be applied to a definition. Either a name has no definite meaning, or we have failed to discover that meaning, if our definition includes things to which the name does not apply or excludes things to which it does.

The object of definition being to explain what a thing is, it is clear that we defeat our own object, if we use language which is as unintelligible as the name of the thing we wish to define (*ignotum per aeque ignotum*), or if we attempt to explain the name by itself or by any other term which implies an acquaintance with it (*circulus in definiendo*), or if we use a metaphor about the subject, which is merely to say what it is like, or if we confine ourselves to saying what it is not.

The second rule, which provides for clearness in a definition, may seemingly be violated when it is not really so. For a definition may be correct enough from a special point of view, which, apart from that particular context, would appear absurd. From the point of view of conic sections, it is correct enough to define a triangle as that section of a cone which is formed by a plane passing through the vertex perpendicularly to the base,

but this could not be expected to make things clearer to a person who was inquiring for the first time into the meaning of the word triangle.

The third rule, which guards against there being anything superfluous in a definition, is a precept of perfection, since a definition may serve all practical purposes without the observance of it.   But as brevity is the soul of wit, so is it also of a good definition.

Again the fourth rule, against defining in a circle, may be violated in appearance without being violated in reality. Thus Euclid, or rather his translator [1], defines an acute-angled triangle as 'that which has three acute angles.' This seems a glaring violation of the rule, but is perfectly correct in its context ; for it has already been explained what is meant by the terms 'triangle' and 'acute angle,' and all that is now required is to distinguish the acute-angled triangle from other kinds of triangle.

The fifth rule, against defining in a circle, is violated, not only if we define a thing by itself, but also if we define it by its contrary, as 'Light is the absence of darkness,' or by its correlative, as 'A ruler is one who has subjects under him.'

The sixth rule cannot always be observed.   For there are many terms which, though positive in form, are privative in force.   Such terms serve as residual heads under which to throw everything within a given sphere

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[1] Euclid's own words are ὀξυγώνιον δὲ τὸ τὰς τρεῖς ὀξείας ἔχον γωνίας, 'an acute-angled triangle is that which has its three angles acute.'

which does not exhibit certain positive attributes. Of this unavoidably negative nature is the definition which we gave of Accident, which amounted merely to saying that it was any attribute which was neither a difference nor a property.

§ 109. There are two ways in which a definition of a thing may be given

(1) by detailing its primary and essential attributes, e. g. when we have enumerated the attributes of a yellow, shining colour, of solidity, heaviness, hardness, of ductility, fusibility, fixedness, and solubility in *aqua regia*, we are considered to have defined 'gold;'

(2) by giving a genus and a difference, or differences [1], e. g. figure is the limit of a solid.

Here figure is referred to the genus 'limit,' of which the other species are the line and the point, which are the limits of the surface and of the line respectively. From these figure is differentiated by being the limit 'of a solid.'

§ 110. A definition serves the practical purpose of enabling us mentally to distinguish the thing defined from all other things. This is not all that a definition does, but, if it fails to do this, it is not a definition. Now it may at first sight seem an endless task to distinguish a given thing from everything else in the world, but there is a short cut to the desired end. If we distinguish the thing defined from the things which it is most like, much

[1] On definition by genus and differences the classical reader may see Aulus Gellius, IV. 1.

more shall we have distinguished it from things which it is less like. This is just what is effected by giving the genus and the difference.

If we were asked to define a triangle, we would not begin by distinguishing it from a hawser, but from a square and other figures with which it is more possible to confound it. The class into which a thing falls is called its Genus, and the attribute or attributes which distinguish it from other members of that class are called its Difference.

§ 111. Care should be taken that the genus chosen is the proximate genus. Thus in defining ' square ' we ought not to refer it to the class ' figure,' but to the nearer class ' quadrilateral or four-sided figure.'

§ 112. The highest class of all, which we express by the word ' thing,' cannot be defined, because there is no genus above it to which it can be referred. It also baffles definition for another reason, namely, that the attribute connoted by it, that of pure existence, is absolutely simple.

§ 113. Definition is an analysis of the subject in intension. It is breaking up a concept into the simpler concepts which compose it. In order then for a thing to admit of definition, the idea of it must be in some way complex. Names of simple attributes defy definition, but at the same time do not require it. We know what is meant by such attributes as redness, sweetness, pleasure, pain, likeness, existence, but we are unable to define them. To a man who has never enjoyed sight, no language can convey an idea of the greenness of the grass or the blueness of the sky ; and if a person were unaware of the mean-

ing of the term, 'sweetness,' no form of words could convey to him an idea of it. We might put a lump of sugar into his mouth, but that would not be a logical definition. Definition cannot, any more than reasoning, be pushed indefinitely backwards.

The art of giving a good definition is to seize upon the salient characteristics of the thing defined and those wherefrom the largest number of other attributes can be deduced as consequences. To do this well requires a special knowledge of the thing in question, and is not the province of the mere logician.

We have seen already, in treating of the Heads of Predicables (§ 69), that the difference between genus and difference on the one hand and property on the other is wholly relative to some assumed definition. Now definitions are to a certain extent arbitrary, and will vary with the point of view from which we consider the thing required to be defined. Thus 'man' is usually contrasted with 'brute,' and from this point of view it is held a sufficient definition of him to say that he is 'a rational animal.' But a theologian might be more anxious to contrast man with supposed incorporeal intelligences, and from this point of view man would be defined as an 'embodied spirit.'

In the two definitions just given it will be noticed that we have really employed exactly the same attributes, only their place as genus and difference has been reversed. It is man's rational, or spiritual, nature which distinguishes him from the brutes : but this is just what he is supposed

to have in common with incorporeal intelligences, from whom he is differentiated by his animal nature.
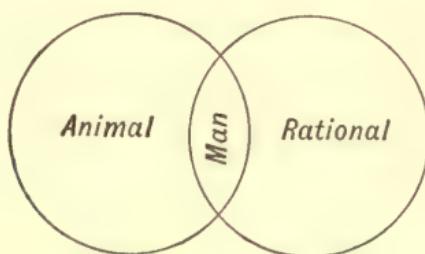
This illustration is sufficient to show us that, while there is no absolute definition of anything, in the sense of a fixed genus and difference, there may at the same time be certain attributes which permanently distinguish the members of a given class from those of all other classes.

§ 114. Definition is confined to abstract terms and to concrete common terms which can be used as subjects. An attributive does not admit of definition, because it has no meaning out of predication ; but it may be defined through the corresponding abstract term, e. g. ' merry ' through ' mirth.'

A concrete singular term, i. e. the name of a primary substance, does not admit of definition, because no one attribute can be considered more essential to an individual than another.   In the case of a class the essential attributes are, or are found among, those which are common to all the members : but in the case of an individual we have no such criterion.   To John as John it is no more essential that he possesses reason than that his hair is red.   If the concrete singular term however be of the kind known as

a designation, it may be defined through the common term which is used in making it up. When we say 'The present Pope is the priest who is at this moment the recognised head of the Roman Catholic Church,' we have fixed the meaning of the term 'present Pope' by superadding to the definition of Pope the attribute of contemporaneous existence. The fact is that such a term is not really singular, though, as there is only one 'now' at any given moment, it is not applicable to more than one thing at a time.

§ 115. There is a sense in which even proper names admit of definition. For in so far as a word has a meaning that meaning can be set forth. Thus we can say 'John is the name of a male person.' If it be objected that John may be the name of a dog, cat, monkey, ship, or institution, the answer is that most words are equivocal, and that if we wish to define any term, we must choose one sense of the same word. Such a definition however is not of the kind of which we have hitherto been speaking. It is not a 'real' but a 'nominal' definition. It does not expound the nature of the being known as John, but only the meaning of the word 'John' before it has been applied to a given individual.

§ 116. This brings us to the recognised distinction between Real and Nominal Definitions.

> A Nominal definition only explains the meaning of a name ;
>
> A Real definition does this by determining the nature of the thing denoted by it.

Hence there may be a nominal definition of a thing which has no real existence.

The definition of a stag or of a goat is a real definition.

The definition of a ' goat-stag ' is a nominal definition.

§ 117. As definition has to do with the intension of terms it is more easy to apply it to abstract than to concrete terms. The reason of this is that the force of abstract terms lies wholly in their intension. The kind of abstract terms that are most easy to define are mathematical conceptions, such as ' triangle,' ' square,' ' circle,' which belong to the region of necessary truth.

Such things have no accidents. If a triangle appears to have accidents such as size and position, it is only because we confuse the abstract concept triangle with some sensible representation of it.

In concrete terms on the other hand the extension is uppermost, not the intension. This is what makes it so hard to define them. We have no difficulty in recognising the things that are denoted by such names as ' animal,' ' plant,' ' chair,' ' book,' but we have a very dim idea of the attributes connoted. For the name gets extended from one thing to another like it until at last there are perhaps no attributes common to the things which it denotes. But the definition of a class must be found somewhere among the attributes which are common to all its members. Hence such names have ceased to be class names at all, and have to be recalled from their vagueness by a new definition, just as defaced coin has to be called in and restamped.

§ 118. The classes of which concrete common terms are the names are either made by nature or by man. Those which are made by nature are called 'real kinds,' e.g. man, dog, cat, pig. The members of such classes agree in various attributes which have no discoverable connexion with one another, and some of which must therefore be regarded as inseparable accidents in relation to those which are selected as constituting the definition. Such classes admit of real propositions being made about them, e.g. 'All cats have whiskers.' The same thing is true of the genera under which these species fall in a natural division, e.g. we can say 'All ruminant animals have cloven hoofs.' On the other hand no such propositions can be made when a class is purely arbitrary. We can make a class of green or white things, but of the members of such a class we can predicate nothing but their greenness or whiteness, unless it be some attribute of these attributes, e.g. of green things that they are restful to the eye and of white things that they are hurtful.

Apart from the difficulty of determining the more or less essential nature of attributes which are united without being connected, nature's classes do not admit of being satisfactorily defined owing to the great law of continuity. They shade off into one another by such imperceptible degrees that it is impossible to fix the boundary-line. Who shall tell us exactly where the plant ends and the animal begins? Under such circumstances there is no single attribute, much less any group of attributes, common to every individual thing to which the name is applied.

The best we can do is to lay down a type of the class in its most perfect form and include or exclude individuals according as they more or less conform to it.

§ 119. From definition must be distinguished Description.

Definition is the analysis of a concept.

Description is the setting forth of the mental picture which accompanies some concepts.

Definition is an appeal to thought.

Description is an appeal to imagination.

Hence description is applicable primarily to individuals, e.g. ' John Smith is the tall man with sandy whiskers who lives next door on the right-hand side and passes by every morning at nine o'clock ; ' secondarily to the classes which lie nearest to individuals and of which we can form some definite mental picture, e.g. ' the Skye terrier is from four to six inches high and from twelve to fourteen long, it is usually either grey or blue-black with dropping ears, and long silky hair, which in perfect specimens altogether covers the eyes [1].'

Description serves equally well with definition to distinguish the subject from all other things, but it accomplishes that object by an enumeration of accidents with or without the mention of some class-name.

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[1] From *A Discourse on Truth*, by Richard Shute, M.A., p. 70. It is given by the author as an instance of a ' definition.'

CHAPTER IX.

Of Division.

§ 120. Logical division consists in breaking up a genus into its component species.

§ 121. From this it is manifest that only common terms can be divided, and also that the members of a division must themselves be common terms.

§ 122. An 'individual' (ἄτομον) is so called as not admitting of logical division. We may divide the term 'cow' into classes, as Jersey, Devonshire, &c., to which the name 'cow' will still be applicable, but the parts of an individual cow are no longer called by the name of the whole, but are known as beefsteaks, briskets, &c.

§ 123. Whether common terms be abstract or concrete makes no difference as regards the possibility of dividing them, since we may classify attributes as well as substances.

§ 124. The term divided is called the Divided Whole (*Totum Divisum*); as given to be divided, it may be called the 'dividend.' The classes into which it is split up are called the Dividing Members (*Membra Dividentia*).

§ 125. A class of which the members were absolutely homogeneous could not be divided. This is the case for practical purposes with coins of the same type and date, as issued from the same mint. Division is only possible by difference.

The first thing then that is requisite in making a division

is to find some attribute in which some members of a class differ from others. This attribute is called the Basis of the Division (*Fundamentum Divisionis*). After coins have been put in circulation, we may find such a basis in the important distinction between *meum* and *tuum*. This is an external and superficial principle on which to divide coins: it is not as coins that they are so divided, but as property. But it is important to notice that the basis of a division must always be sought in some separable accident of the class divided. If we were to base a division on the difference of the divided whole, or on one of its properties, or even on an inseparable accident, all the individuals denoted by the name would be found in a single compartment of the division, and the other compartments would be left empty. Thus if we were to divide Kangaroos into Australian kangaroos and others, we should find all the kangaroos on one side of the division.

§ 126. Division, like definition, varies with the purpose in hand. The same class may be divided on different bases, e. g. we may divide animals according to the element which they principally inhabit, according to the number of their legs, according to the mode in which they produce their young, according to the nature of their food, and so on indefinitely. Again, men may be divided on the basis of colour, or on that of locality, or on that of race or language or sex or temperament: it all depends on the point of view which interests us at a given moment. A good division follows the natural lines of cleavage : it is a process of carving, not of hacking.

§ 127. *Rules for Division.*
I. Main Rules.

(1) The division must be conducted on a single basis.

   Violation. Churches into Gothic, episcopal, high, and
      low.

(2) The basis must be a separable accident of the
genus.

   Violation. Men into rational and irrational.

(3) The species, when added together, must be co-
extensive with the genus.

   Violations. Triangles into right-angled and acute-
      angled.

   Coins into gold, silver, bronze, and bank-notes.

The effect of violating the first rule is to produce what
is called a ' cross-division.'

The effect of violating the second rule is to have the
form of division without the matter.

The effect of violating the third rule is twofold—

   (*a*) that the division is not exhaustive [1],

   (*b*) that it is redundant.

The above rules are all that are necessary, but the
following will be found useful in testing a division and
distinguishing it from other processes which are something
like it.

§ 128. II. Additional Rules.

(4) The term divided must be predicable of each of
the dividing members.

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[1] Cum praeterire aliquid maximum vitium in dividendo sit.
Cic. Off. I, § 10.

Violation.   Horse into cart-horse, pony, mule, and
   donkey.

(5) Each member of the division must be a common
term.

Violation.   Great Britain into England, Wales, and
   Scotland.

(6) The dividing members must be mutually exclu-
sive, i. e. no individual must find a place under more than
one of them.

Violation.   Man into philosopher, red-haired, Greek,
   and slave.

These additional rules are corollaries from the main
rules and from the conception of logical division.

(4) If the genus be not predicable of each of the
species, it is plain that the species, when added to-
gether, cannot exactly make up the genus.

(5) If each member of the division be not a common
term, we run counter to the definition from which we
started.

(6) If the division be conducted on a single basis, the
constituent species must exclude one another.   The con-
verse however does not always hold true.   We may have
a division consisting of mutually exclusive members which
yet involves a mixture of different bases, e. g. if we were
to divide triangle into scalene, isosceles, and equiangular.
This happens because two distinct attributes may be found
in invariable conjunction.

The sixth rule may be violated either by the total or
partial inclusion of one of the members in the other.

The first will be the case, if a genus be co-ordinated with its own species, if, for instance, we were to divide the parts of speech into noun, article, pronoun, substantive, adjective, verb, and particle. The true division is into noun, verb, and particle, the rest being sub-divisions of noun. When the inclusion of one member in the other is only partial, the species are said to overlap, as in the example previously given. An individual like Epictetus may very well have found a place in all four compartments of 'philosopher,' 'red-haired,' 'Greek,' and 'slave.'

In testing a division the sixth rule is the best to begin on. If there be any individuals that would fall under two or more heads, we may be sure that two or more different bases have been mixed in the division. If man, for instance, were to be divided into European, Asiatic, Aryan, and Semitic, the species would overlap; for both Europe and Asia contain inhabitants of Aryan and Semitic origin. We have here a division based on locality, mixed up with another based on race as indicated by language.

§ 129. As the differences between species are separable accidents of the genus to which they belong, it is plain that division involves an appeal to experience. Prior to experience we have only the form of division without the matter.

We know for instance that men are either white or not-white, but we do not know whether there are any white men or any but white men. Thus the only

purely formal division is a hypothetical division by dichotomy.

Here is a series of such divisions.

```
                        Man
            ┌────────────┴────────────────┐
      White (if any)              not-white (if any)
                           ┌─────────────┴─────────────┐
                     Black (if any)            not-black (if any)
                                          ┌──────────────┴──────────────┐
                                    Brown (if any)            not-brown (if any)
                                                         ┌──────────────┴──────────────┐
                                                    Red (if any)            not-red (if any).
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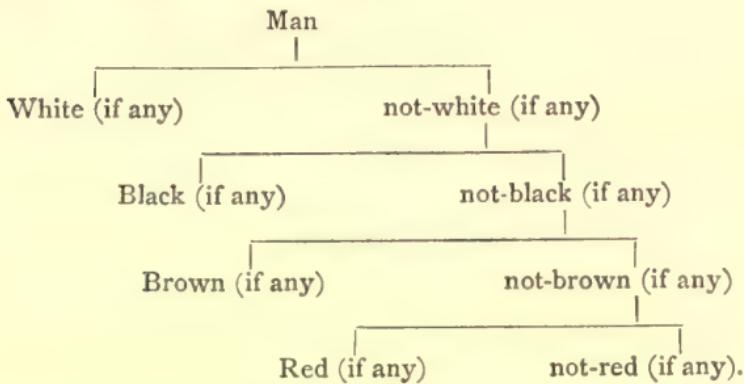
Division by Dichotomy means the division of a genus into two species, one of which possesses a given attribute while the other does not[1]. It is division by means of a pair of contradictory terms. Hence it is plain that one side in such a division must always be negative.

Experience assures us that for ‘not-red’ men we may substitute ‘yellow,’ and so arrive at the fivefold division of man on the basis of colour—

```
                        Man
      ┌───────┬─────────┼─────────┬──────────┐
    White   black     brown      red      yellow.
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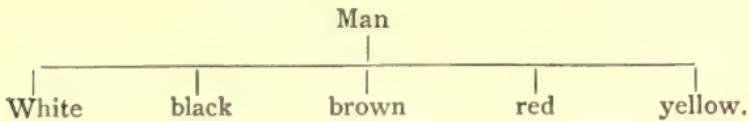
§ 130. Any correct logical division in which there are more than two species is the compressed result of a series of divisions by dichotomy. Thus, to take another example, the term quadrilateral, or four-sided rectilinear

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[1] Called by the Stoics ἀντιδιαίρεσις (Diog. Laert. VII, § 61).

figure, is correctly divided into square, oblong, rhombus, rhomboid, and trapezium. The steps of which this division consists are as follows —

```
                        Quadrilateral
                             |
           ┌─────────────────┴─────────────────┐
           |                                   |
     Parallelogram                         Trapezium
           |
     ┌─────┴─────────────────────┐
     |                           |
  Rectangle                 Non-rectangle
     |                           |
  ┌──┴──┐                    ┌───┴────┐
  |     |                    |        |
Square Oblong            Rhombus   Rhomboid.
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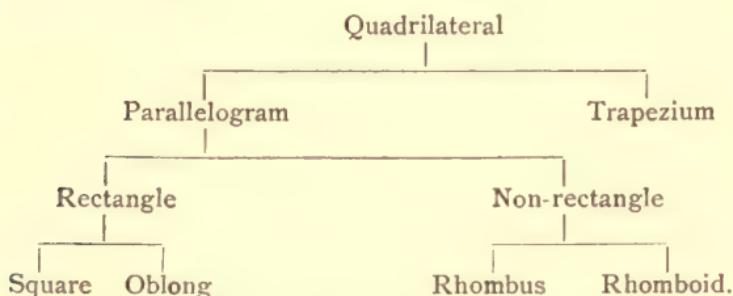
§ 131. In such a scheme of division and sub-division [1] the Summum Genus is the highest class taken; the Infimae Species (lowest kinds) are the classes at which the division stops.

Subaltern Genera and Species are the classes intermediate between the highest and lowest, which are genera in relation to the classes below them and species in relation to the classes above them.

Cognate Species (kindred kinds) are those which fall immediately under the same genus; Cognate Genera are the classes under which a lower class successively falls, e. g. rectangle, parallelogram, and quadrilateral are cognate genera of square.

The relation of cognate species to one another is like that of children of the same parents, whereas cognate genera resemble a line of ancestry.

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[1] Diog. Laert. VII, § 61—Ὑποδιαίρεσις δέ ἐστι διαίρεσις ἐπὶ διαιρέσει.

The Specific Difference of anything is the attribute or attributes which distinguish it from its cognate species, e. g. of a square that its sides are equal. A specific difference is said to constitute the species. The Generic Difference of anything is the difference of its proximate genus, e. g. of a square that its angles are right angles. A Specific Property of anything is an attribute which follows from its specific difference, e. g. of a square as opposed to an oblong, that it is divisible into two isosceles triangles. A Generic Property of anything is an attribute which follows from its generic difference, e. g. of a square that it is half the size of the square on its own diagonal.

The rule that the division must be conducted on a single basis applies only to a single act of division. When we go on to subdivide, we not only may, but must, adopt a new basis, since *ex hypothesi* the old one has been exhausted. Thus, having divided men according to the colour of their skins, if we wish to subdivide any of the classes, we must look out for some fresh attribute wherein some men of the same complexion differ from others, e. g. we might divide black men into woolly-haired blacks, such as the Negroes, and straight-haired blacks, like the natives of Australia.

What is called the divided whole with reference to a single act of division becomes the *summum genus* where there is subdivision. Similarly the *infimae species* correspond to the dividing members of a single act of division.

In reckoning up the *infimae species* care must be taken

not to include any class which has already been sub-divided[1], e. g. in the division of 'quadrilateral' rectangle must not be mentioned along with square and oblong. No harm is done however by mixing an undivided class, like trapezium, with the subdivisions of its cognate species; the latter represent the higher class, as children under a will represent the deceased parent.

§ 132. Enumeration is the statement of the individual things to which a name applies. Like division, it is an analysis of a term in extension, only that it is carried further. As in division, the name is predicable of all the members, e. g. 'month' of January, February . . . December.

§ 133. Partition is a statement of parts, not of kinds, e. g. when we say that a tree consists of roots, stem, branches, twigs, and leaves, or that a proposition consists of subject, predicate, and copula. Partition is not con-fined to physical division; indeed its chief use is in rhetoric.

In a partition the name of the whole is not predicable of each of the parts.

§ 134. Distinction is the separation from one another of the different meanings of an equivocal word. In a distinction the name only is predicable of each of the members, but the definition is in each case different.

§ 135. Metaphysical division is the analysis of a sub-

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[1] Cicero, *De Inv.* I, § 33 *ad init.*—'hoc igitur vitandum est, ne, cuius rei genus posueris, eius, sicuti aliquam diversam rem ac dissimilem, partem ponas in eadem partitione.'

stance into its attributes, e. g. of a marble into roundness, solidity, hardness, &c., of man into animality and rationality.

A metaphysical whole must be distinguished from a logical whole.

A Logical Whole is a whole in extension.

A Metaphysical Whole is a whole in intension.

Considered extensively, species is part of genus.

Man + brute = animal.

Considered intensively, genus is part of species.

Animal + rational = man.

§ 136. The following classification of inferences will illustrate division and subdivision and at the same time serve as a map of the country which we have next to explore.
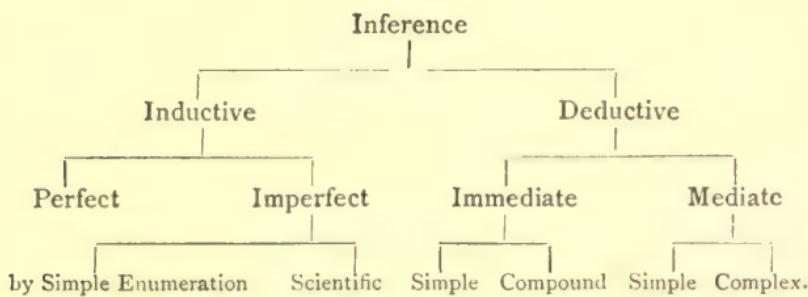
PART III.

Of Inferences.

CHAPTER I.

Of Inferences in General.

§ 137. In the widest sense of the term we are said to 'infer,' when we arrive at some truth, not by direct experience, but as a consequence of some truth or truths already known. If you see a door-step strewn with rice, you do not 'infer' that there is rice on the door-step—you see that; but you will probably infer that there has been a wedding from the house, and be right in your inference. Why? Because of your previous experience of a happy couple being pelted with rice when they leave a house on their honeymoon. If this experience has already formed in your mind the general proposition 'Wherever the door-step is strewn with rice, there has been a wedding,' your inference is deductive; if this general proposition is now being formed, your inference is inductive. But however you arrive at your conclusion, you would equally be said in common language to 'infer.'

Inference then, in the sense of inferring, is the passage of the mind from one or more propositions to another.

An inference is the result of inferring.

§ 138. Every inference consists of two parts—

    (1) the truth or truths already known, which are called the Antecedent;

    (2) the truth which we arrive at therefrom, which is called the Consequent.

    N.B.—Antecedent and Consequent are here applied to the two parts of an inference; previously they were applied to the two parts of a complex proposition.

      The word 'inference' is sometimes applied to antecedent and consequent together; sometimes it is used of the consequent alone.

§ 139. Inferences are either Inductive or Deductive.

The antecedent in an inductive inference may consist of one, two, or any number of propositions; in a deductive inference it consists of one or of two propositions, according as the inference is Immediate or Mediate.

In the mediate inference, or Syllogism, the two propositions which form the antecedent are commonly called 'the premisses,' and this term is sometimes extended to the antecedent in an inductive inference. The consequent is in all cases a single proposition, and is otherwise known as 'the conclusion.'

Inductive Inference leads to general truths (ἐπὶ τὰς ἀρχάς); deductive inference starts from them (ἀπὸ τῶν ἀρχῶν). That must suffice for the present to distinguish the two processes, until we have recognised the strong difference that there is between 'induction' in the ancient and in the modern sense.

§ 140. Since inferences, whether inductive or deductive, are combinations of propositions, while propositions are combinations of terms, it is evident that the first two parts of logic are equally necessary as an introduction to the study either of induction or of deduction.

§ 141. But under the third part of logic it is natural to treat of inductive before deductive inferences, since it is induction which supplies us for the most part with the major premisses from which deduction starts. In the ordinary course of things the climbing of a mountain must precede the descending of it on the other side, but we may have been born on the top, or have been deposited there by a balloon, or, like Noah, by a flood. Even so the universal propositions, which are required for deduction, are usually arrived at by the toilsome road of experience, but they may be supplied to us by intuition or hypothesis or authority.

§ 142. The meaning which has been assigned above to the word 'inference' is not that either of Sir William Hamilton or of John Stuart Mill, who have been the principal legislators for logical terminology in Great Britain.

Inference with Sir William Hamilton consists in 'the carrying out into the last proposition what was virtually contained in the antecedent judgments' (*Lectures on Logic*, XV, § 54). This excludes a real induction from being called 'inference' at all. For in a real induction the last proposition, or conclusion, must be more than the sum of the particulars from which it is derived.

On the other hand, Mill declares that all inference, and consequently all knowledge of truths not self-evident, rests upon induction. He will not allow that there is any inference except when we pass 'from the known to the unknown.'

Thus Sir William Hamilton's definition of inference excludes induction, while Mill's excludes deduction from coming under the term. In adopting a definition which is wide enough to include both views, we seem to have the further advantage of being more in accordance with common language than either.

CHAPTER II.

Of Perfect Induction and the Inductive Syllogism.

§ 143. Inductive inference is the process from particular [1] to universal truths. It is of two kinds—perfect and imperfect.

In a Perfect Induction all the particulars are examined ; in an Imperfect Induction only so many as are held to be sufficient, perhaps no more than one. Imperfect induction in its highest form is called scientific induction.

The two kinds of process appeal to quite different evidence. Perfect induction rests its validity on a complete enumeration of the instances ; scientific induction on the uniformity of nature. Perfect induction does not introduce the idea of cause ; scientific induction is founded on that idea.

Perfect Induction.

§ 144. Perfect induction consists in asserting or denying of a whole class what has already been found true of every member.

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[1] 'Particular' as here used means simply not-general. It includes both particular and singular propositions.

When we have satisfied ourselves that the names Stephen, Philip, Prochorus, Nicanor, Timon, Parmenas, and Nicolaus are Greek, we may sum up the result by saying 'All the seven deacons had Greek names.'

§ 145. Perfect induction is not limited to the enumeration of individual instances. We make a perfect induction whenever we predicate any attribute of a genus on the strength of its being found in every species. The truths from which we start are in this case particular as compared with the conclusion.

§ 146. The scientific value of perfect induction lies in the fact that it substitutes a compendious for a prolix statement. It is much easier to say that it has rained all the week than to say that it has rained on Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday.

The Inductive Syllogism.

§ 147. Perfect induction may be thrown into the form of a syllogism, thus—

$$X, Y, Z \text{ are } A,$$
$$X, Y, Z \text{ are } B,$$
$$\therefore \text{ All B is A;}$$

where X, Y, Z represent all the individual members of a species or all the species of a genus. We shall see later that this argument violates the rules of the syllogism, and yet it is perfectly valid. The reason of this is that the rules of syllogism are not designed to meet the case of a quantified predicate such as we have in the second

proposition. When we say 'X, Y, Z *are* B,' we mean that they make up B. We may therefore substitute B for them. This will appear more clearly from a concrete instance.

> Matthew, Mark, Luke, and John were Jews.
>
> Matthew, Mark, Luke, and John were the four Evangelists.
>
> ∴ The four Evangelists were Jews.

§ 148. Perfect induction from this point of view may be regarded as the syllogism reversed. Read the propositions backwards and we get a set of syllogisms proving the particulars from which we started in the induction.

> The four Evangelists were Jews.
>
> Matthew (or Mark or Luke or John) was one of the four Evangelists.
>
> ∴ Matthew (or Mark or Luke or John) was a Jew.

In the syllogism we argue that what is true of all is true of each.

In perfect induction we argue that what is true of each is true of all.

CHAPTER III.

Of Imperfect Induction.

§ 149. In a perfect induction the universal conclusion at which we arrive is no wider than the sum of the particulars.

In an imperfect induction the conclusion is extended beyond the cases examined, so as to include all like cases.

The process from the part to the whole, from the less to the more general, can never be a certain one, but there are different degrees of hazard attending it.

Induction by Simple Enumeration.

§ 150. Induction by Simple Enumeration consists in affirming something as a universal truth on the mere ground of uncontradicted experience, without any attempt to arrive at a cause. If one draws marbles from a bag, and the first six prove to be green, one expects the seventh to be so. On this ground all swans were in ancient times asserted to be white, and even now there may be forests in Africa where all men are believed to be black. This is the induction which Bacon[1] con-

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[1] *Novum Organum*, Lib. I, Aph. cv.

demned as 'a childish thing, precarious in its conclusions, and exposed to risk from a contradictory instance.' But, notwithstanding its childishness, it has great influence over adult intelligences. Hume indeed found in the conception of Cause itself nothing but this power of custom to mould the mind.

Scientific Induction.

§ 151. Scientific Induction is an attempt to arrive at laws of causation by detecting the real connexion of things which underlies superficial connexions.

The connexion between the swan-nature and white is a superficial connexion, as was shown by the discovery in Australia of black swans, which the Ancients took as typical of the prodigious—

> Rara avis in terris, nigroque simillima cygno.

But if it could be duly established by observation and experiment that there was something in the climate of Europe, and not in that of Australia, which blanched the feathers of a swan, we should then have a scientific induction, valid as far as it went, though doubtless not penetrating to the ultimate fact of causation.

§ 152. The specific difference between an imperfect and a perfect induction has come of late to overshadow in importance the difference between induction generally and deduction ; so that it is now often said that perfect induction is a form of deductive inference. When people

speak thus, the point whereby inductive inference is differentiated from deductive is this—

In Induction we proceed from the less to the more general; in Deduction we do not.

The same thing may be expressed more technically thus—

In an inductive inference the consequent is wider than the antecedent; in a deductive inference it is not.

This is an exhaustive division of inferences, being arrived at by dichotomy.

§ 153. If therefore induction be given as the sole alternative for deduction, it will not do to say that in induction we proceed from the less to the more general; in deduction from the more to the less general. This is like forgetting that there is such a thing as walking on a flat surface as well as ascending or descending a hill. For the consequent may be neither more nor less general than, but just as general as, the antecedent. Still less admissible is it to say that in induction we argue from the particular to the universal, in deduction from the universal to the particular. For, in addition to what has just been said, the propositions on which an induction is based may themselves be universals, which are merged in a statement of still higher universality.

Traduction.

The name Traductive Inference is sometimes applied to the case in which the consequent is just as wide as the antecedent. Under this head come the inductive syllogism,

most forms of immediate deductive inference, and syllogisms made up of singular propositions, e. g.

> Brutus was the founder of Roman liberty.
> Brutus was a patrician.
> ∴ A patrician was the founder of Roman liberty.

Traductive inference, it will be observed, is a species of deductive inference.

CHAPTER IV.

Of Apparent Inductions.

§ 154. LET it be understood that by 'induction' we mean henceforward what is known as 'real' or 'scientific' induction. We have seen already that perfect induction is not in this sense an induction at all. In this chapter we shall follow Mill in distinguishing other cases of apparent from real induction.

§ 155. When a mathematician has proved that a straight line cannot meet a circle in more than two points, and has then established the same thing successively of the ellipse, the parabola, and the hyperbola, does he make an induction in laying this down as a universal property of the sections of the cone? The answer is 'No! there is no process from the known to the unknown.' Mill's reason for distinguishing this from an ordinary instance of perfect induction is that the conclusion here is a true generalisation, although it adds nothing to the facts, the cases dealt with being demonstrably all the ways in which it is possible for a cone to be intersected by a plane. This however does not affect the form of the argument, which, like any other perfect induction, falls under the head of traduction.

§ 156. Again, when we have proved of the particular triangle ABC that its three angles are equal to two right angles, do we make an induction in asserting the same of all triangles? If we do, says Mill, 'an appropriate name would be induction by parity of reasoning.' But he is of opinion that we do not. For though we infer a universal conclusion when we have only examined a particular instance, the conclusion nevertheless is not believed on the evidence of the particular instance. We believe it because we perceive that the process of demonstration which is applied to one case might equally well be applied to all. This is how Mill himself answers the question; but those who, in spite of Berkeley, still retain a belief in abstract ideas would say that the demonstration never referred to a particular triangle at all, but to the idea of a triangle, or to the concept three-sided figure. There is therefore in this case no room for generalisation. When, as in the seventh proposition of the first book of Euclid, there are different cases to be considered, we do make a generalisation, when we assert as the final result that it is impossible in any case that there should be two similar triangles on the same side of the same base. But here again the generalisation adds nothing to the facts, and the argument falls under perfect induction.

§ 157. Again a mathematician, after having calculated a sufficient number of terms to arrive at what is called the *law* of the series, does not hesitate to assume that the subsequent terms will resemble the preceding. Nevertheless this cannot properly be called induction. For

his conviction of uniformity, in order to be valid, must rest on *a priori* considerations; and, when these have not been arrived at, the assumption of uniformity has sometimes proved fallacious. As an instance of how we may be deceived by trusting to uniformity in arithmetic, take the following series of numbers—

$$1,\ 7,\ 31,\ 127.$$

Each of these is an odd power of 2 diminished by 1.

1st power	$2-1=$	1
3rd ,,	$8-1=$	7
5th ,,	$32-1=$	31
7th ,,	$128-1=$	127.

And each term in the series is a prime number. What more tempting then than to lay down the law that any odd power of 2 diminished by 1 will give us a prime number? Yes: but try the next term of the series. The ninth power of 2 diminished by 1 is 511, which is divisible by 7. We have here then only the precarious induction by simple enumeration upset by a contradictory instance.

§ 158. The last process which Mill distinguishes from true induction is what was called by Dr. Whewell 'the Colligation of Facts.' The well-worn instance, employed both by Whewell and Mill, will serve to show the nature of this process. Kepler wished to discover what kind of orbit the planet Mars moved in. It was impossible to observe its progress continuously; but a number of detached observations had been made as to the apparent places occupied by the planet. The question for Kepler

was what kind of orbit the spaces between these points would make, supposing them to be all joined together. After a good deal of unsuccessful guessing Kepler discovered that an ellipse exactly suited the case. Now here, says Mill, there was no induction, but merely a description in general terms of a set of observed phenomena. For there was nothing in the conclusion which was not already contained in the observations on which it was based. We might as well say that a navigator, after sailing round some newly-discovered land, makes an induction when he declares it to be an island.

The term 'Colligation of Facts' is associated in the minds of readers of Mill's *Logic* with a somewhat tedious controversy. Mill accuses his opponent of confounding it with induction; and that this charge is not unjust may be gathered from the following passage in the *Philosophy of the Inductive Sciences* (Vol. I, p. 43)—'The discovery of a truth by induction consists in finding a conception or combination of conceptions which agrees with, connects, and arranges the facts.' But at the same time a Colligation of Facts, according to Dr. Whewell, is more than the mere sum of them, since in it there is introduced a mental conception which is not in the facts themselves. 'The facts are known, but they are insulated and unconnected, till the discoverer supplies from his own store a principle of connexion. The pearls are there, but they will not hang together till some one provides the string.' To this Mill answers, fairly enough, that it is true the *conception* is not in the facts themselves, but that the

properties conceived of are; and that, if the conception
be a correct one, it must be a copy of something in the
facts. Sometimes however, owing to difficulties of obser-
vation, the conception is not derived directly from the
facts contemplated, but the mind has to supply various
conceptions till one is found to fit them. The latter, Mill
acknowledges, was the case with Kepler. Nevertheless
the fitting conception can only be that which describes the
actual state of the facts. The selection of conceptions
may be called guesswork, but such guessing is the gift of
genius.

The upshot of the matter seems to be that induction is
something more than finding a conception to fit certain
facts. Every induction indeed is a colligation of facts—
it is a form of finding the One in the Many—but every
colligation of facts is not an induction. Here, for instance,
are certain facts in the Latin language. Among masculine
and feminine nouns with nominative in *-or* only *arbor* is
feminine ; all the rest are masculine. These facts taken
as they stand are arbitrary. We reduce them to law
when we find that those which have a long vowel in
the stem are masculine, while the only one which has
a short vowel is feminine. This may be said to be the
fitting conception whereby we colligate the facts. But
there is no induction here, for there has been no discovery
of a cause.

CHAPTER V.

Of the Axioms of Induction.

§ 159. HAVING seen sufficiently what induction is not, it is now time to attend to what it is.

Induction is the process by which we arrive at general truths by the way of experience.

If induction is to begin from the beginning, it must start from particular facts.

These particular facts become known to us by Experience. Experience is either external or internal.

External experience is the use of the senses either on the part of ourselves or of others.

If each human being were to shut himself up within the limits of his own experience, very little progress in knowledge would be possible. By far the greater number of the truths that we suppose ourselves to know come to us from the senses of other people. This use of the senses at second-hand is called Testimony. This is the source of all that great department of knowledge which is called History. Science has to depend largely upon it, but seeks to bring everything to the test of immediate perception.

Internal experience is the consciousness of what passes within our own minds when we feel, think, and will. This is the ultimate source of that other great department

of human knowledge, or rather speculation, which is called Philosophy.

§ 160. But here a difficulty presents itself at the outset. Experience, strictly speaking, only refers to the present; by the aid of memory it can be extended to the past: but it is silent as to the future. Again experience only testifies to what is immediately before it: it has nothing to say as to the innumerable things and events even in the present which are beyond its ken. If experience is thus limited to what is *here* and *now*, how can we by its aid divine what is elsewhere or what is yet to be? How, in other words, can we ever reach general truths by the aid of experience? Experience can assure me that this fire burns and this water wets. But how am I to know that the next fire I meet may not cool or the next water crush me? The question sounds nonsensical, but that is because we judge of it after repeated experience, when we have by some means or other arrived at two general truths, namely, that fire burns and water wets.

By what means then were these general truths arrived at? Not by any elaborate process of reasoning. The method was a very simple one. We took them for granted on the first experience, and Nature has ever since been verifying our hypothesis. Not the burnt child only, but the burnt puppy also, dreads the fire. The inductive instinct is rooted deep in our animal nature, and without it no intelligence that possessed a body could have preserved that body in a universe like ours. They would all have shattered their organisms long ago in

a blind collision with natural forces. It is owing to the inductive instinct that, when once a connexion has been established in our minds between two things, such as fire and burning, we assume that this connexion holds in all cases and that what is fact once is fact always. This inductive instinct is not infallible. It oftens sets up hypotheses which Nature fails to verify. An eclipse may be followed in one instance by a national calamity, or a comet by a good vintage, but it does not follow that all eclipses and all comets will have the same attendant circumstances. The hypotheses that are not verified have to be dropped by the race as well as by the individual.

Observe then that in the very possibility of arriving at general knowledge by experience there are two assumptions involved—first, that there is a connexion between events, secondly, that this connexion will continue. These two assumptions convert our world into a rational system. Let us for a moment refuse to make them. Let us suppose ourselves transported from the causal world in which we live into a purely casual world. In such a world events would take place anyhow, and always on their own account, without the least connexion with what went before or what followed after them. Chance and spontaneity would be the order of the day. If contact with fire happened on a given occasion to be followed by the pain we call burning, we should say that the pain arose of itself without any connexion with the antecedent circumstance. Under such a system the only knowledge possible would be a register of particular facts. These

particular facts would furnish no clue to other cases, and we should never rise to a general proposition. It is only because we live in a world in which antecedents and consequents have a fixed relation that general knowledge is possible to us.

But now let us make the first of these assumptions without the second. Let us suppose that there is a connexion between events, but deny that this connexion is bound to continue. Should we be any nearer the possibility of general knowledge than before? If the water which wet us once were to bite us another time instead, or the fire which burnt us yesterday were to entertain us with a concert to-day, we should still be involved in all the perplexity of a casual universe. The causes, if we chose to call them so, in such a world would resemble capricious human wills, whose action could not be relied upon. Any belief we might hold as to causation in a particular case would be an hypothesis which Nature failed to verify. It would be as reasonable to hold that burning took place of itself as that fire burnt us in a single instance, if fire acted differently again.

§ 161. When the two assumptions above spoken of are clothed in philosophical language, they assume the form of the two axioms of induction, which are known as—

> (1) The Law of Causation.
> (2) The Law of Uniformity.

The Law of Causation lays down that

> ' Every event has a cause.'

It denies that things take place of their own accord or spontaneously; but whenever an event happens, directs us to something that went before, as having brought it to pass.

The Law of Uniformity asserts further that—

' The same cause always produces the same effect,' or, in other words, that under like circumstances like results ensue.

§ 162. When these axioms have once been laid down, all inductive inference is turned into deductive. What was an animal instinct that could give no account of itself now assumes the dignity of reasoning. Every inductive inference, as Archbishop Whately declared, may be regarded as a syllogism with the major premiss suppressed; and this major, when supplied, lays down that what holds true of the cases under consideration will hold true of all similar cases. Thus when, having found that this, that, and the other magnet attract iron, we advance to the general proposition, ' All magnets attract iron,' our reasoning may be expressed as follows—

> Whatever holds true of this, that, and the other magnet holds true of all.
>
> Attracting iron holds true of this, that, and the other magnet.

∴ Attracting iron holds true of all.

Now the assumed major here is not self-evident. How then do we come by it? It is plain that it rests for its evidence on the larger assumption of uniformity in the course of nature. While the immediate major premiss

then in every instance of induction is the affirmation of
some particular kind of uniformity, the ultimate major
premiss is the Law of Uniformity itself. The real problem
of induction is to ascertain when we are warranted in
asserting that what holds true of one case holds true of
all. In order to solve this problem we have to eliminate
all the unessential circumstances until we arrive at the
pure fact of sequence which underlies the appearances.

§ 163. Mill admits that 'every induction may be thrown
into the form of a syllogism,' and that, when this is done,
the principle ' of the uniformity of the course of nature
will appear as the ultimate major premiss of all induc-
tions' (III. 3, § 1): but, in accordance with his restriction
of inference, and along with it of proof, to induction,
he maintains that the major premiss does not at all con-
tribute to prove the conclusion, though it is a necessary
condition of its being proved; since no conclusion is
proved for which there cannot be found a true major
premiss—in other words we are never warranted in
drawing a particular except on the same evidence as
would suffice for a universal. But if we subtract the
major premiss from the proof, what is there left but the
inductive instinct which leads us from one particular case
to another like it? This may be called reasoning, if we
admit that brutes reason: but who will call it proof?
It might better however to say that human reason rests
upon this animal intelligence than that this animal in-
telligence is itself reason. When this brute instinct
realises itself in man, it becomes converted into faith

in the uniformity of nature—a faith as essential to the scientific as faith in God to the religious mind. Without the one we can find no spiritual order, without the other no physical order, in the universe.

§ 164. Reverting now to the first of our two axioms, let us be careful not to put more meaning into it than it properly conveys. The Law of Causation asserts that 'Every event has a cause.' It does not assert that every thing has a cause. There must be an uncaused somewhere, put it where we will, whether in matter or in mind, or in something behind and above them both.

CHAPTER VI.

Of Observation and Experiment.

§ 165. EXPERIENCE may be divided into two branches—

> (1) Observation.
> (2) Experiment.

Observation is passive experience.

Experiment is active experience.

In observation we study things under nature's conditions.

In experiment we impose conditions of our own.

The object of both is to furnish us with true facts on which to base our reasoning. Hence the extreme importance of the two processes, since they form the foundation of the temple of knowledge. The faculty for looking facts in the face is much rarer than might be imagined. A teacher is often called upon to combat difficulties which arise merely from the student misreading a book which lies open before him. The mistake is generally traceable to some prepossession in his mind. So it is with the book of nature. All sorts of baleful phantoms obsess the mind of man and prevent it from seeing things as they are.

§ 166. Experiment is a more potent engine of discovery than observation. The following are the chief advantages which it possesses over the sister process—

(1) We can isolate the phenomenon.

(2) We can vary the circumstances.

(3) We can reproduce the phenomenon at will.

Again experiment is more precise, more subtle, and more expeditious than observation. It is more precise, because we can surround the phenomenon under investigation with the exact set of circumstances which we require. It is more subtle, because it enables us to detect small changes, which might escape mere observation. It is more expeditious, because it can give us in a short time what nature might present to us only after a long period, or might never present to us at all.

On the other hand we may say that observation has two advantages over experiment—

1. It can be applied to many branches of knowledge where experiment is out of the question. For instance, we can observe the courses of the stars, but we cannot experiment upon them, as if they were so many billiard balls. Again we can observe what issue results from the intermarriage of a lunatic and an idiot, but it would be highly improper to experiment in this direction.

2. It can be used in the search for causes as well as in the search for effects, whereas experiment, from the nature of the case, is limited to the latter. We can take a cause and experiment as to its effect: we cannot take

an effect and experiment as to its cause ; for ignorance
of the cause, as Bacon says, deprives us of the effect.
All that we can do then is to watch till we see the effect
produced, unless indeed we produce it by accident.

§ 167. There is however no essential difference between
observation and experiment ; the one runs into the other ;
it is a question of degree, not of kind. Even the most
passive observation involves the mental act of attention.
A meteorologist is said to take observations at the foot
of a mountain ; but if he carries his barometer up to the
top, that amounts to an experiment, for he is then varying
the circumstances.   The separating things that usually
go together and the combining things that are usually
found separate is the great object of experiment.   If this
is in any case done for us by nature, the result is as
productive of light as if we had done it for ourselves.

§ 168. What we call 'observation' is a process con-
sisting partly of sensation and partly of inference.   When
Berkeley sat down to write his Essay towards a New
Theory of Vision, he took it for granted as 'agreed by
all, that distance of itself, and immediately, cannot be
seen.'   By distance he meant distance in the line of sight.
It was agreed then even in his time that a large part
of what we suppose ourselves to see—all in fact that
relates to a third dimension of space—is really inferred.
The novelty of his theory consisted only in the new
ground on which he declared the inference to be based.

These inferences which enter into perception by the
senses are instinctive but not infallible.   I remember

as a boy lying on the grass on a summer's day looking up into the sky. I saw, as I thought, a very big bird very high up in the air. In another moment I found that I was mistaken and that what had really happened was that a very small fly had passed quite close to my eye. I had somehow drawn a wrong inference from the sensation actually experienced.

The large range for inference that there is in observation is what makes it not unphilosophical for us to distrust within due limits our own and other people's senses. Even a passing thought in the mind may serve at times to make us read into the facts what is not in them. For instance, some years ago as I was coming down to breakfast I heard a noise in my study and, the door being partly open, I saw, as I imagined, inside the room a Parsee who was staying in the house with me, with the cap on his head which he usually wore. When he came down to the dining-room, I referred to his having been in the study, and he assured me that he had only that moment come down from his bedroom. I have no doubt that what I really caught sight of was the housemaid dusting the room, and that some untraceable current of thought led me to impose on her the appearance of the Parsee. Again here is an experience which is not uncommon with me and which I do not suppose is peculiar to myself. I see some one whom I know advancing towards me in a crowded street. On getting nearer I find that I am mistaken, but very soon afterwards I see the real person who had been in my mind. Perhaps this fact may be

accounted for thus—that really I have seen the person in the distance without attending to the impression, and that this impression then fixes itself on the wrong individual. At all events I hold that as a working hypothesis.   I have never known the experience occur when the real person was round a corner.

CHAPTER VII.

Of Uniformities of Coexistence.

§ 169. INDUCTIVE inference is mainly concerned with the relation of cause and effect, because it is concerned with the establishment of general truths, and things as a rule do not hold true generally unless there be a cause why they should do so. I say 'as a rule,' because there are some general truths that do not rest upon causation. Truths of causation are uniformities of succession; but besides these there are uniformities of coexistence.

§ 170. Let us begin with a humble instance from our own science. It is a general truth that exactly six moods are valid in all the figures, yet no reason, so far as I am aware, can be assigned for the fact. Each figure indeed has its special rules and we can show that there are six moods which do not break any of them. As this is the case with each, a perfect induction assures us that it is so with all. But that all should coincide in the same number is a mere piece of undesigned symmetry. All that we can say is, it so happens that they do. But suppose some one were to supply us with a reason for believing that there must be the same number of moods valid in each figure, this would not make the truth to be one of causation.

For a reason is not a cause, though it is often confounded
with it. A reason is something that belongs to the region
of deduction, not of induction. It is in fact a premiss or,
when stated fully, the premisses which prove some con-
clusion.

§ 171. The general truths with which the purely mathe-
matical sciences deal are uniformities of coexistence, not
of succession. The different properties, for instance, of
the triangle and the circle can be shown by reasoning, or
appear by intuition, to inhere in the same concept; but
there is no question of causation involved. Inductive
inference has nothing to do with them, because they are
not arrived at by the way of experience.

§ 172. Coming then to physical truths, to which the
notion of cause alone applies, we may notice in the first
place that there are certain uniformities of coexistence
which are directly traceable to causation, as being joint
effects of a common cause, e. g. the coexistence of light
and darkness in the opposite hemispheres of our globe—

> Nosque ubi primus equis Oriens afflavit anhelis,
> Illic sera rubens accendit lumina Vesper.

There are also many cases of coexistence in which we
may suspect causation but cannot prove it, as in the
attributes possessed in common by natural kinds. Nobody
can show a cause why animals that chew the cud should
divide the hoof: but there stands the fact for all that.
It rests only upon an induction by simple enumeration.

The union of mind with body, and of mind in its higher
manifestations with a complex structure of brain, is a law

of coexistence to which no exception has as yet been generally admitted: disembodied intelligence is not recognised by science, nor do we ever find a lump of metal entertaining us with its wit and wisdom, whatever amount of 'mind-stuff' there may be latent in it.

The widest of all laws of coexistence is the connexion between the two attributes of body known as inertia and gravity—a connexion which, within the limits of our experience, is found to pervade all nature. 'Inertia' is a rather misleading term. It has to cover, not only the refusal of bodies to move, unless force be applied to them, but also their refusal to stop moving, unless some counteracting cause be brought to bear upon them. Now that gravity is an attribute distinct from inertia may be seen from the fact that gravity, or the attraction of matter for matter, is itself one of the counter agents to the continued motion of a projectile. This force at last brings it to earth, where its motion is soon stopped by the resistance of that body. And yet these two attributes, though thus distinct, are invariably found together, and in exact proportion to one another. It is not, properly speaking, the weight of a heavy body which prevents your pushing it along a level surface, but the inertia which is proportional to the weight.

§ 173. The truths with which the grammarian and the student of language deal are uniformities of coexistence, not of succession. Let us take an instance from Greek Grammar. By a wide employment of the Method of Agreement it may be shown that the only common element

in all forms of the optative mood is the single letter iota.
But when our inductive survey is complete, we do not say
that iota is the *cause* of the optative, but only that it is the
sign of the optative. A sign is something quite different
from a cause. The same cause has always the same
effect, but the same sign may have different significates.
Thus, though iota is the sign of the optative, that does not
prevent it from being the sign of the dative too [1].

We cannot apply the Method of Difference to signs,
since they involve, at least directly, no causation. Nor in
the particular case chosen can we employ the Method of
Concomitant Variations, since neither iota nor the force of
the optative is a variable quantity : but we can employ the
Method of Residues and argue from the known force
of the other formative suffixes that the special force of the
optative must be signified by the iota. Here however we
are anticipating, as the inductive methods have yet to be
explained. We will therefore quit this subject with the
remark that, though a great deal has been done for the
logic of succession, very little attention has been bestowed
upon the logic of coexistence, which seems still to await
its Bacon or its Mill. This is perhaps due to the fact
that it rests solely upon Agreement, and has therefore
been deemed of minor importance.

[1] Hume (*Essays*, Vol. II, p. 64 n, ed. Green and Grose) dis-
tinguishes between Cause and Sign thus—'A *Cause* is different
from a *Sign* ; as it implies Precedency and Contiguity in Time
and Place, as well as constant Conjunction. A Sign is nothing but
a correlative Effect from the same Cause.'

CHAPTER VIII.

Of the Conception of Cause.

§ 174. Since it is the relation of cause and effect that supplies us with most of the general truths which we reach by the way of experience, and since induction consequently is chiefly concerned with this relation, it becomes of importance to attain, if we can, a right conception of cause. In order to do so, we must begin by defining some other words.

§ 175. First let us take the word 'phenomenon.'

As used in common life 'phenomenon' means something out of the ordinary course of nature. An eclipse, a comet, or an aurora borealis is much more readily called a phenomenon than a sunrise or sunset. When we see an announcement of the 'infant phenomenon,' we do not expect an ordinary baby. This connexion with the marvellous however must be entirely rejected from the meaning of the term as used in logic.

Again in ordinary parlance 'phenomenon' tends to be confined to an appearance to the eye. But in the scientific use of the term the peal of the thunder is no less a phenomenon than the flash of the lightning, the smell of a rose no less than the colour, the feel of furniture in the dark no less than the 'sparks' which result from knocking one's eye against it. In short a phenomenon is anything which presents itself to the senses.

Here we might stop, if there were none but physical phenomena to be taken into the account. But people speak also of mental phenomena. We must therefore enlarge our definition and say that a phenomenon is anything which is presented in experience either external or internal [1].

§ 176. Now, given any phenomenon—say, a twinge of gout in somebody's big toe—the whole history of the universe may be divided into the events which follow that phenomenon and the events which do not. All that follow it are called Consequents; all that do not are called Antecedents. Thus every phenomenon serves as a landmark of history, dividing it by a sharp dichotomy.

Let the reader notice that we have not attempted to divide the course of events into those which go before and those which come after a given phenomenon. This would be to leave out all the events contemporaneous with it. At the moment when somebody felt that twinge of gout in his big toe there were innumerable events happening in every quarter of the globe and throughout the solar system and in the abysses of space and in states of being beyond our ken. All these are, properly speaking, Concomitants of the given phenomenon: but, for

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[1] The word 'phenomenon' seems to have been pitched upon with a view to expressing the facts from which we start in our inductive inquiries without raising any metaphysical questions as to a reality underlying them. It is consequently a very wide and vague term, confusing as it does under one head material substances and the sensations into which we resolve them, together with all states of consciousness, and all events, which are relations between substances.

convenience of expression, the accompanying are merged with the preceding circumstances under the one head of Antecedents.

An Antecedent then may be defined as anything which precedes or accompanies a given phenomenon; and a Consequent as anything which follows a given phenomenon [1].

§ 177. Now among the innumerable antecedents of any phenomenon there are some that have a certain bearing upon its production and others that have none. Preceding the supposed twinge of gout there was all that has happened in our universe from the time of the solar nebula and whatever may have gone before that down to the nineteenth century. But we do not regard all these events as contributing to the phenomenon in question. In a certain sense indeed it would be very difficult to draw the line between contributary and non-contributary antecedents. For, if we remember that gout implies a body, it becomes evident that all the changes on the earth's surface which rendered possible the existence of animal organisms are at once brought in. But this way of looking at the thing would make any such speculation an endless task. So we cut short the inquiry by a postulate—Given a human body, what are the antecedents which have some bearing upon the production of a twinge of gout? The proximate cause is the presence of uric

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[1] The words 'antecedent' and 'consequent' are very hard worked. This is the third time that we have had to define them in different senses. See § 138.

acid in the blood.   But what are the antecedents of this?
Let us say that they may be summed up under the two
heads of inherited tendencies in the constitution and a too
free indulgence in good living.   Neither of these by itself
would produce the phenomenon : but, if either were taken
away, the phenomenon would not occur.   The inherited
tendencies might be there, and yet gout never be de-
veloped, if a careful rule of living were observed ; while
on the other hand the too free indulgence in good living
on the part of a person not of gouty constitution, though
it would have other bad effects, would not produce the
particular phenomenon which we have in view.   Now we
want a term to separate off the antecedents connected with
the phenomenon from those which are unconnected with
it.   The term which does this is ' condition.'   A Condition
then we define as an indispensable antecedent of a pheno-
menon, or, that without which something would not be.

§ 178.   The word ' cause ' is popularly used for the
particular condition which happens to be uppermost in
the mind at a given moment or in some special con-
nexion.   Thus the cause of an explosion by gunpowder
might be said to be the properties of the gunpowder, or
the setting fire to it by knocking the ashes out of a pipe,
or the culpable carelessness of the man who smoked near
it.   Philosophically speaking, however, the cause is not one
but all of these.

Again let us suppose that a labourer has been killed by
a fall from a ladder.   The British jury returns a verdict
of ' accidental death.'   But let us look into the case more

in detail. The accident took place in the morning; there had been rain the night before, and the rungs of the ladder were wet; the man was carrying a hod of bricks on his shoulder; his foot slipped on one of the rungs, so that the weight got shifted; he lost his balance and down he came. The overseer might in his own mind set down the wetness of the rungs to be the cause of the death and be more careful in future to have the ladders kept under cover; a mechanician might attribute the disaster to the shifting of the weight and the consequent loss of equilibrium; while the physical philosopher might see in it an instance of the attraction of matter for matter. But now let us supply a detail which failed to come before the jury. The man's wife had accompanied him to his work that morning. On the way they had had a speculative difference. The wife with a parting shot had emphasized her view of the case, which caused her husband to look round before he was safe on the scaffolding, whereupon the accident ensued. The poor widow might for the rest of her life regard her own unseasonable argumentative-ness as the sole cause of the phenomenon, to the exclusion of the part played by lubricity and gravitation. Her speaking however was not the *cause*, but rather the *occasion* of her husband's death. By an Occasion is meant the change which brings the other parts of the cause into operation. It is the spark which lights a train of gunpowder. Luther's burning of the Pope's Bull was not the cause, but might be said to have been the occasion of the Reformation.

While the term 'occasion' may thus be employed for some salient and positive incident which completes the collocation of circumstances required for the production of a given effect, nothing but the whole collocation of circumstances can in the full sense be considered the cause.

We therefore now reach a definition of Cause as the sum of the conditions of a phenomenon, or, that, given which, something is, or takes place. Conversely an Effect may be defined as something which always is, or takes place, when a given set of conditions is fulfilled.

§ 179. In another sense, which it is impossible to avoid, the word 'cause' is used for any substance regarded as an agent, as when we say that fire is the cause of heat or that a man struck a blow. It is in this sense we use the term when speaking of the

Combination of Causes.

When two or more causes acting together produce more of the same kind of effect which each would have produced singly, we have what is known as Mechanical Combination of Causes or Mechanical Intermixture of Effects, e. g. two horses pulling a load. When two or more causes acting together produce a different kind of effect from what any of them would have produced singly, we have what is known as Chemical Combination of Causes or Chemical Intermixture of Effects, e. g. oxygen and hydrogen producing water.

Mechanical combination of causes is also called by Mill the Composition of Causes. The joint effect of mechanical causes is the same as the sum of the separate

effects, even when it least seems to be so. Thus when two equal forces acting in opposite directions keep a body at rest, this is the same as though one had pushed it so far in one direction and then the other had pushed it back ; and again when two forces acting upon a body at right angles propel it in a diagonal line, this is the same as though the body had reached the same point by describing two sides of a square under the action of the forces separately.

On the other hand the joint effect of chemical causes is not calculable from that of the separate effects, but requires a fresh appeal to experience. This is why chemistry is not a deductive science.

Mechanical intermixture of effects is also called Homogeneous.

Chemical intermixture of effects is also called Heterogeneous or Heteropathic.

Mechanical intermixture is the rule, chemical the exception. All objects in some respects obey the law of composition of causes. However startling the transformations which substances undergo through chemical combination, the weight of the compound is always found to be the precise equivalent of the weight of the elements.

§ 180. The maxim that effects are proportional to their causes applies only to the case of homogeneous intermixture of effects, for where effects are heterogeneous there can be no proportion between them. Neither is this maxim applicable to the apparent cause known as ' the occasion.' One spark igniting gunpowder is quite as effective as two.

CHAPTER IX.

Of the Inductive Methods.

§ 181. THE Inductive or Experimental Methods are the ways which the mind naturally adopts of establishing by aid of experience the effect of a given cause or the cause of a given effect. They are both methods of discovery and methods of proof, but it is under the latter aspect only that logic has to deal with them.

§ 182. These methods are five [1] in number.

    (1) The Method of Agreement.

    (2) The Method of Difference.

    (3) The Indirect Method of Difference.

    (4) The Method of Concomitant Variations.

    (5) The Method of Residues.

183. These methods may be applied either to proving causes or to proving effects, but for simplicity their Canons will here be stated only from the former point of view.

Canon of the Method of Agreement.

If several instances of the occurrence of a phenomenon agree only in one thing, that thing is probably the cause or part of the cause.

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[1] Mill speaks of four methods and gives five. It seems to be the Indirect Method of Difference which he does not consider entitled to rank independently. Chapter 8 of Book III. is entitled 'Of the Four Methods of Experimental Inquiry.' Sometimes Mill speaks of the 'Inductive Methods' (III. 9, § 6).

Canon of the Method of Difference.

If an instance of the occurrence and an instance of the non-occurrence of the same phenomenon differ only in one thing, which is present in the former and absent in the latter, that thing is the cause or part of the cause.

Canon of the Indirect Method of Difference.

If several instances in which a phenomenon occurs agree only in one thing and several instances different from the former, in which it might be expected to occur but does not, agree in the absence of the same thing, that thing is probably the cause or part of the cause.

Canon of the Method of Concomitant Variations.

When two phenomena vary together, they are connected by some tie of causation.

Canon of the Method of Residues.

If from a whole effect there be subtracted such parts as are due to known causes, the residue will be the effect of the unknown cause or causes.

Illustration of the Method of Agreement.

Antecedents.	Consequents.
A B C	a b c
A D E	a d e
A F G	a f g.

Let 'a' be an effect, of which we wish to establish the cause.

The argument assumes the following form.

Neither B, C, D, E, F, nor G is the cause of 'a,' for it is present where they are not.

But 'a' must have a cause.

∴ A is the cause.

This argument is not conclusive, owing to what is known as Plurality of Causes [1].

It has been laid down that the same cause always produces the same effect, but it cannot be inferred from this that the same effect is always produced by the same cause. The same effect, death, for instance, may be due to such different causes as starvation, thirst, suffocation, disease, accident, violence, or decay.

Now it is possible that the effect 'a' is due in the first instance to B or C, in the second to D or E, in the third to F or G, and that A, though the only antecedent common to all the cases, had nothing to do with it. If a doctor were to cure the same disease on three separate occasions by three different drugs administered each time in orange wine, it would not follow that orange wine was a grand specific for that form of malady, though the non-medical observer might jump to the conclusion that it was.

In the possibility of a plurality of causes lies the

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[1] The fact of plurality of causes has often been denied. Hume (*Treatise of Human Nature*, Part III, Sect. xv. 4) lays down the axiom 'that the same effect never arises but from the same cause.' Reid, who differs from him in so much else, agrees with him so heartily here as to say that 'the axiom upon which all our knowledge of nature is built' is 'that effects of the same kind must have the same cause' (*Inquiry into the Human Mind*, chap. vi, sect. 24).

inherent weakness of the Method of Agreement. It is always conceivable, though not probable, that the same effect may be produced by a different cause in each of the cases we are considering, and that some purely irrelevant circumstance may at the same time be present in them all. The more instances we collect, the less likely this becomes. Number of instances then is of value in this method, in order to exclude the chance of coincidence.

The Method of Agreement is chiefly a method of observation. It can point to laws of sequence, but, apart from experiment, we cannot be sure that they are laws of causation. Its chief use indeed is in suggesting applications of the Method of Difference.

§ 185. *Illustration of the Method of Difference.*

Antecedents.	Consequents.
A B C D	a b c d
B C D	b c d.

Let 'a' be an effect, of which we wish to establish the cause.

The argument assumes the following form.

The same cause always produces the same effect.

Now B, C, and D occur in the second instance without being followed by 'a.'

∴ They are not the cause.

But 'a' must have a cause.

∴ A is the cause.

The reasoning here is conclusive. The practical difficulty is to make sure that we have got two such

instances as are required.   Nature seldom presents us
with them : we have to contrive them for ourselves.   The
Method of Difference is therefore the great method of
experiment.

A room is quite dark ; you press a button and it is
flooded with light.   Why do you believe that the pressing
of the button had something to do with the production
of light ?   Because you find no other difference between
the circumstances of the room at the one moment, when
it is in darkness, and at the next, when it is in light.
It would be a very elliptical way of speaking to call the
pressing of the button the *cause* of the light, but it is
a good instance of what we have called the *occasion.*   In
a room unsupplied with electric apparatus you might
press a button to the end of time without any production
of light.   But suppose that there were no accessory cir-
cumstances and nothing intermediate between the single
antecedent of pressing a button and the consequent light,
we should then be justified in saying that the light was
caused by the pressing of the button.   When Gyges
found that he became invisible, he ascribed the cause to
the collet of his ring being turned inwards, because that
was the only difference discoverable between the moment
when he was visible and the moment when he was not
(Plat. *Rep.* 359, 360).

The great difficulty in an experiment is to make sure
that there is no hidden antecedent.   Sometimes the effect
is due, not to the change itself, whether positive or
negative, which we introduce into a set of known

conditions, but to the means taken to bring it about. If there is doubt on this point, it has to be settled by other experiments.

Ideally the Method of Difference is perfect, but, owing to the complexity of nature, we can never be absolutely sure that its conditions are complied with.

§ 186. *Illustration of the Indirect Method of Difference* [1].

Antecedents.	Consequents.
A B C	a b c
A D E	a d e
A F G	a f g
B H	b h
D K	d k
F L	f l.

Here the probable conclusion reached in the positive instances by the Method of Agreement, that A is the cause of 'a,' is confirmed by the negative evidence of the absence of 'a,' where A is not.

It must of course be understood that the negative instances come within the same range of fact as the positive ones; otherwise the absence of the given antecedent would be of no significance. This is indicated in the illustration by the retention of one of the old

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[1] An alternative name for this method proposed by Mill himself is the Joint Method of Agreement and Difference. He also speaks of it as 'a double employment of the Method of Agreement' (III. 8, § 4). Hence Professor Bain suggests the name Method of Double Agreement (*Induction*, p. 61) and Professor Fowler has entitled it the Double Method of Agreement.

antecedents in each of the negative instances.   This is also the reason why in the statement of the canon (§ 183) we inserted the words 'in which the phenomenon might be expected to occur but does not.'

Further it should be noticed that the negative instances must differ from the positive in more points than the mere absence of A.   If, for instance, instead of B H . . . b h we had B C . . . b c, the comparison of this with A B C . . . a b c would give us the direct Method of Difference, and all the other instances would be superfluous.   This is why in the statement of the canon we inserted the words ' different from the former.'

If the negative instances are really to the point, our presumption that A is the cause of ' a ' is greatly strengthened.   Still the argument does not reach the demonstrative certainty which is attained, at least formally, by the Method of Difference.   It would only do so, if the negative instances were so contrived as to eliminate all the antecedents other than A [1].

Antecedents.	Consequents.
A B C	a b c
A D E	a d e
A F G	a f g
B E	b e
C F	c f
D G	d g.

[1] On this point I am indebted to Mr. Charles Cannan of Trinity College, Oxford.   He claims no monopoly, in the idea, and my only wish is to indicate the source from which it came to me.

Here we may argue as in the Method of Difference itself.

The same cause always produces the same effect.

Now B, C, D, E, F, G occur in the second set of instances without being followed by 'a.'

∴ They are not the cause.

But 'a' must have a cause.

∴ A is the cause.

The above is the ideal type of the Joint Method. It is therefore fitly called the Indirect Method of Difference. Instead of proving our point directly by a comparison of two instances we attain the same end more circuitously by a comparison of several.

In practice however the Indirect Method of Difference has to be very roughly used. Doctors satisfy themselves by this method that vaccination prevents small-pox. They compare a number of vaccinated with a number of unvaccinated districts, and point to the absence of small-pox in the former and its presence in the latter. By an employment of the same method Berkeley satisfied himself that tar-water had the same effect. He tried it in his own neighbourhood, when the small-pox raged with great violence. 'And the trial,' he says, 'fully answered my expectation: all those within my knowledge who took the tar-water having either escaped that distemper, or had it very favourably. In one family there was a remarkable instance of seven children, who came all very well through the small-pox except one young child which could not be brought to drink tar-water as the rest had done' (*Siris*, § 2).

The Indirect Method of Difference is sometimes regarded as a double application of the Method of Agreement—

(1) to instances in which the phenomenon occurs,

(2) to instances in which it does not occur.

But the Method of Agreement is really inapplicable in the latter case, since the negative instances must agree in the absence of innumerable antecedents.

§ 187. *Illustration of the Method of Concomitant Variations.*

Antecedents.	Consequents.
12A BC	6a bc
10A BC	5a bc
8A BC	4a bc.

Let A be an effect of which we wish to establish the cause. We obtain instances of it which vary in amount or degree. Then, if among the antecedents we find one, and only one, which varies correspondingly, we come to the conclusion that there is a causal connexion between the two.

If we could be certain that there were no antecedents with any bearing on the effect except those before us, we could say that A is the cause of 'a,' since the cause must be looked for among the antecedents. The argument would then assume the following form.

No constant factor can be the cause of changes in an effect.

B and C are constant factors.

∴ B and C are not causes of changes in 'a.'

But the changes in 'a' must have a cause.

∴ The changes in A are the cause of them.

The major premiss laid down above, that 'No constant factor can be the cause of changes in an effect' is only a disguised form of the axiom that 'No event takes place without a cause.' For, if an effect were to vary while the cause remained constant, we should then have a certain event, namely, a change in the effect, which took place without a cause.

But the assumption that we have all the conditions before us is too hazardous to be made. Accordingly we leave it open to be supposed that both A and 'a' are joint effects of some common cause. For instance, the hour hand and minute hand of a watch present us with a case of concomitant variations, but we do not suppose the movements of one to cause those of the other : they are both caused by the works within. This illustration may also serve to show that this method may be used without any analysis into antecedents and consequents. The influence of the moon on the tides and the connexion between the barometer and the weather are obvious instances of concomitant variations. We are able in both these cases to distinguish correctly between antecedents and consequents. But suppose a savage to see a barometer in the house of a European and to become aware of the fact that the mercury went up and down with the changes in the weather, his inference would probably be that he was in the presence of some potent rain-maker.

Statistics are often an application of this method.   Let us take an imaginary example.

A village in 1890 has 5 public-houses and there are 100 cases of drunkenness in the year.

In 1892 it has 4 public-houses and there are 80 cases of drunkenness in the year.

In 1894 it has 3 public-houses and there are 60 cases of drunkenness in the year.

These figures might seem to show that public-houses are the cause of drunkenness, but the decrease of both may be due to some moral reform in the locality or merely to a decline in population.

It should also be noticed that the two phenomena may vary inversely as one another.   In this case it is the absence of one that is causally connected with the presence of the other.   The length of the days in summer is not the cause of the shortness of the nights, nor conversely ; but both phenomena depend upon the same cause, namely, the position of the earth with regard to the sun at that season.

From one aspect the Method of Concomitant Variations is an approximation to the Method of Difference. In the Method of Difference we entirely remove a certain antecedent ; in this method we observe the effect of its diminution.   It is useful in the case of causes that cannot be wholly got rid of, e. g. the attraction of the earth, the heat of bodies.   It may also be applied with advantage after the Method of Difference, to give quantitative precision to its results.

§ 188. *Illustration of the Method of Residues.*

Antecedents.	Consequents.
A B C D	a b c d
B C D	b c d.

To the eye this is the same as the Method of Difference. The distinction between the two methods lies only in the way in which the negative instance is arrived at. In the Method of Difference it is got by immediate experience ; in the Method of Residues it is the result of previous knowledge. This however makes no difference to the reasoning [1].

This potent instrument of discovery and proof is no more than a sum in subtraction. It comes into use in science after a certain advance has been made in the observation of phenomena and the study of causes. But it is of course a method of reasoning familiar to us all in our daily life. Thus, suppose a man to be black-balled by a society consisting at the moment of twelve members : if he knows that ten were prepared to vote for him, he ascribes his exclusion to one or other of the remaining two. Or suppose that some Sunday morning in a poor country parish there appears the phenomenon of a half-sovereign in the offertory; the clergyman knows by repeated experience that none of his flock ever by any chance give more than a silver threepenny, but he

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[1] It is not really necessary in this method for the negative instance to be stated at all.

has perceived a stranger present in the congregation—
does he hesitate to regard him as the cause of the
phenomenon?

§ 189. In further illustration of the methods let us now
answer a question put by Jevons [1]—

What can you infer from the following instances?

Antecedents.					Consequents.
ABDE	.	.	.	.	. stqp
BCD	.	.	.	.	. qsr
BFG	.	.	.	.	. vqu
ADE	.	.	.	.	. tsp
BHK	.	.	.	.	. zqw
ABFG	.	.	.	.	. pquv
ABE	.	.	.	.	. pqt.

Starting this time from the cause and searching for the
effect, we will first try what can be done by the use of
the Method of Agreement alone.

In the four cases in which A occurs the consequents
are stqp, tsp, pquv, pqt. A more orderly arrangement
of these makes it easier to see that the one thing common
to them all is p—

> pq st
> p   st
> pq   uv
> pq t.

We therefore infer that the effect of A is p.   In the

same manner the student may assure himself that the Method of Agreement yields altogether these results—

$$A \quad . \quad . \quad . \quad . \quad . \quad . \quad p$$
$$B \quad . \quad . \quad . \quad . \quad . \quad . \quad q$$
$$D \quad . \quad . \quad . \quad . \quad . \quad . \quad s$$
$$E \quad . \quad . \quad . \quad . \quad . \quad . \quad t.$$

About C, HK, and FG it can only tell us what we already see before us.

But the Method of Agreement is at best uncertain in its results. Can we then confirm our conclusions in any way? Yes: by a comparison of these two cases

$$BFG \quad . \quad . \quad . \quad quv$$
$$ABFG \quad . \quad . \quad . \quad pquv$$

we become sure that A is the cause of p.

Altogether by the Method of Difference we can establish these results—

$$A \quad . \quad . \quad . \quad . \quad . \quad . \quad p$$
$$B \quad . \quad . \quad . \quad . \quad . \quad . \quad q$$
$$D \quad . \quad . \quad . \quad . \quad . \quad . \quad s.$$

But we are left uncertain as to E, and wholly in the dark about C.

The Indirect Method of Difference strengthens our previous presumption that E is the cause of t: for where E is, t is; and where E is not, t is not. We cannot rely on it much in the case of C, since there is only one instance of the presence of that antecedent. Still, as far as it goes, it points to C being the cause of r: for r occurs in the one case in which C is, and nowhere occurs where C is not.

But at this point of the investigation we are able to avail ourselves of a more drastic method. In the instance BCD ... qrs we know already by the Method of Difference that B is the cause of q and D of s. Therefore it remains that C is the cause of r. The same Method of Residues enables us also to assert that FG are jointly responsible for uv and HK for wz.

§ 190. A fallacious appearance of simplicity is imparted to these methods by the use of letters, which represent events as already analysed into antecedents and consequents, whereas the whole difficulty of inquiry consists in this process. We are tempted also to think that a cause is a single antecedent, whereas it may be the conjunction of several. Moreover in actual experience effects seldom appear side by side, but are indistinguishably merged in a single sum.

Suppose the phenomenon under investigation to be table-turning. Let us imagine a party of undergraduates assembled in the rooms of a man whom we will call Jones. He is the owner of a small round table which will accommodate three persons, while the whole party numbers seven—Jones himself, Smith, Brown, Robinson, Taylor, Williams, and Thomas. Various sets of three are tried in turn. The experiment is uniformly successful, the table exhibiting surprising physical agility, and even some mental power in the way of answering questions by tilting at a certain letter when the alphabet is recited to it, until it has in this way spelt out a sentence. Next morning in the absence of Jones the rest of the party are discussing

their experience of the evening before. Smith declares that he had nothing to do with the occurrence of the phenomenon, appealing in confirmation of his assertion to the fact that the table was equally lively when he was not at it. In this way one of the antecedents is eliminated. Then the remaining five make the same declaration, all in good faith. All the antecedents are now eliminated except one. It now occurs to our investigators that Jones was a member of every set that was formed. Jones is thereupon convicted in his absence of being the cause of the phenomenon, on the ground that he is the only common antecedent. This conclusion, it will be seen, is reached by the Method of Agreement.

Taylor however, who is mystically inclined, demurs to this conclusion on the ground that all possible causes of the phenomenon are not summed up in the persons of the seven sitters. The solemn question, which has been asked at the *séance*—'Is there a spirit moving this table?' cannot, he declares, ever be answered in the negative. In this view he is theoretically unassailable : nevertheless the question *is* answered in the negative by the more materially-minded of his associates, led by Thomas, who has imbibed from his philosophy-tutor the great principle of Occam—*Entia non sunt multiplicanda praeter necessitatem.*

Williams, who never agrees with any one, if he can help it, expresses an opinion that Jones had nothing whatever to do with the result. He is a bit of a mathematician and has noticed that all the combinations of sitters

other than Jones were not exhausted during the course
of the evening.   Instead of the fifteen sets of three which
were possible with Jones present every time, only twelve
were actually tried.   Having kept an eye on the point,
he is able to assert that the following three sets were
omitted—

Jones	Jones	Jones
Taylor	Taylor	Williams
Williams	Thomas	Thomas.

On the strength of his calculations he starts a theory
of his own.   Jones, it is true, was present every time :
but it is also true that either Smith, Brown, or Robinson
took part in every set that was formed, and any of these,
he maintains, was more likely to have been the cause than
Jones.   He, it will be observed, has hit upon what Mill
calls the 'characteristic imperfection' of the Method of
Agreement, which lurks in the possibility of 'Plurality of
Causes.'

As a result of the discussion our inquirers agree to
meet again in Jones's rooms that evening.

The first set to experiment on this occasion are Taylor,
Williams, and Thomas.   The result is a dead failure.
Those who already suspected Jones of being at the bottom
of the matter are now confirmed in their opinion.   For
on the evening before, when Jones formed part of every
set, the 'manifestations' occurred; now, in his absence
from the table, they do not occur.   This, so far as it goes,
is an application of the Indirect Method of Difference.

After this the following sets are tried—

Jones	Jones	Jones
Robinson	Brown	Smith
Thomas	Williams	Taylor

—and with striking success in each case. But, while the third of these sets is in full career following the gyrations of the table, a happy idea occurs to Thomas. He calls out 'Jones, take your hands off!' Jones complies, and the table stops dead. Here we have the Method of Difference, which, if the experiment is rigorous, proves Jones to be the cause or part of it. Accordingly the partisans of the 'Jones' theory are now triumphant.

Taylor however, who contends for the presence of unknown antecedents, is not satisfied. He denies that Jones is the cause, but admits him to be connected with the cause, regarding him merely as the 'medium,' through whom the 'spirit' operates. But now the party discover that they have reckoned without their host. For Jones bethinks himself that two can play at the game of 'Hands off!' and the first person whom he selects for his experiment is the sceptic Thomas. When Thomas's hands are withdrawn, the table equally comes to a standstill. Flushed with success, Jones continues his researches, until he has brought home the production of the phenomenon to each of the party in turn, on the same evidence on which it was attributed to himself. Here is Plurality of Causes with a vengeance! But Taylor, Williams, and Thomas declare that at least they cannot be the cause,

for the table would not stir for them; and this statement is found to stand the test of repeated experience.

It is now borne in upon the minds of our investigators that the cause is not so simple as they were at first inclined to imagine. They all have something to do with it, and yet the mere fact of any three people having their hands on the table is not enough to produce the phenomenon. Apart then from unknown antecedents, it must be some particular combination of persons that is essential to the result. Eventually they satisfy themselves that, though Jones is the only person whose absence is invariably fatal to the phenomenon, yet his operation is not effectual unless he is supported by Smith, Brown, or Robinson and some third member of the party indifferently. What physical or psychical conditions underlie these surface facts they are unable to divine: but it is something to have got so far, even if they can get no further.

CHAPTER X.

Of the Nature of the Inductive Methods.

§ 191. It is evident that what are called the Inductive Methods are really applications of deductive inference to the facts supplied by experience. They are types, to which if we can get facts to conform, we may be certain of our inferences with regard to them. But to procure this compliance on the part of facts is the real difficulty, since 'the subtlety of nature far exceeds the subtlety of human sense or understanding.' Nothing can be truer than this maxim of Bacon's. But when the Father of Induction goes on to assert that the syllogism *assensum constringit, non res*, is he equally worthy of credence? Not unless by the syllogism he meant the supposed general truths which serve for its major premisses. The cogency of the syllogism is based on this principle—that things must be consistent with themselves. There is therefore no escaping it. If the premisses we start from are in accordance with fact, then the conclusions we arrive at will be in accordance with fact also. Whether the mind constrains things or things constrain the mind or whether both are constrained by something behind them is not a question which concerns the logician as such. He simply

assumes the fact that the laws of thought are laws of things. If they were not, reasoning would be unavailing. The great desideratum then is that our supposed general truths should be really in accordance with fact, and the use of the methods is that they supply us with so many standards by which to try this accordance. Even if the methods can never be strictly complied with, this does not diminish their value as standards.

§ 192. For the Method of Agreement to be applied successfully we must first eliminate the chance of mere coincidence. This however can easily be done by taking a sufficient number of instances. An antecedent which has no connexion with the effect under investigation may be present in one or two instances, but is hardly likely to be present in many. After this the possibility of a plurality of causes must be allowed for, since it cannot be eliminated. But if M and N are alternative causes with A of the phenomenon 'a,' this can be proved in the same way as in the case of A itself.

Suppose however that after any amount, not merely of observation, but of experiments carefully conducted, A is still found to be the sole invariable antecedent of 'a,' are we then warranted in declaring it to be the cause? Not unless we are prepared to assert that all the material circumstances are within our knowledge. But who will be bold enough to make this assertion? Until within the last couple of years chemists imagined that they knew all the constituents of the air we breathe; nevertheless within that period several new ones have been discovered. The

insufficiency however of the Method of Agreement to prove causation is admitted. Let us go on to the Method of Difference.

§ 193. If we can take A and by its means produce ‘a’ at will, who shall deny that the one is the cause of the other? Certainly no one can deny that there is a causal connexion. But how are we to know that ‘a’ is really caused by A, and not by some latent antecedent involved in the taking of A? Let us suppose however that that contingency is guarded against. Even now we cannot be sure that we have reached the pure fact of sequence. Nature is infinitely complex, and what we regard as the single phenomenon A may really be compounded of a, β, γ; of these β and γ may be wholly irrelevant, and it may only be the a in A that is the cause of ‘a.’

§ 194. The Indirect Method of Difference, the Method of Concomitant Variations, and the Method of Residues are all modifications of the Method of Difference. Hence all the methods are reduced by Mill himself to these two— Agreement and Difference. We must either compare together instances in which the phenomenon occurs or else compare instances in which it does occur with instances in which it does not occur: it is of no more use comparing together instances in which it does not occur than it would be to frame a syllogism with two negative premisses. But may we not go one step further and say that there is after all only one method? We prove that one thing is the cause of another by showing that nothing else can be. They are all alike methods of elimination, that is,

of discarding antecedents which are not conditions, or
consequents which are not effects.

'The Method of Agreement stands on the ground that
whatever can be eliminated is not connected with the phe-
nomenon by a law.

'The Method of Difference has for its foundation, that
whatever cannot be eliminated is connected with the
phenomenon by a law' (Mill, III. 8, § 3).

In the former we reach by elimination a single point of
agreement; in the latter a single point of difference.

In the Method of Agreement the cases *differ* in all
respects save one.

In the Method of Difference the cases *agree* in all
respects save one.

As the one point of agreement or of difference is vital,
it is this which gives the name to the method.

§ 195. When Whewell objected against Mill's methods
that they were not the processes whereby discoveries had
actually been made, Mill replied *firstly* that Dr. Whewell's
argument, if good at all, was good against all inferences
from experience, for if any discoveries were made by
observation and experiment, it was by processes reducible
to one or other of those methods; *secondly*, that even if
they were not methods of discovery, it would be no less
true that they were the sole methods of proof. Mill
regarded himself as supplying rules and models for
inductive inference such as the syllogism and its rules are
for ratiocination (III. 9, § 6). Was he not rather rational-
ising the inductive instinct by bringing its conclusions

under the Axioms of Induction? His methods, unless we take them as confined to Agreement and Difference, lack the demonstrative exhaustiveness of the four figures of the syllogism. Nevertheless, despite Whewell's objections, they have won their way to general acceptance and have become part of the common stock of thought. This is because they represent living processes which we are all conscious of employing in the daily exercise of the rational life. But these processes are not independent of deduction. This is admitted by Mill himself (III. 8, § 7) in the case of the Method of Residues, but it is equally true of all. The fact is, Mill's treatment of induction is an important contribution to the fourth part of logic which deals with Method.

CHAPTER XI.

Of some other points connected with Induction.

§ 196. *The Deductive Method.*

The perfect type of scientific reasoning consists neither wholly of induction nor wholly of deduction, but of a due admixture of both. It is called the Deductive Method and consists of three steps—

    (1) Induction, whereby some wide general truth is arrived at.

    (2) Deduction, whereby certain consequences are inferred from this truth.

    (3) Verification, whereby an appeal is made to nature to see whether these consequences actually ensue.

§ 197. *Hypothesis.*

Sometimes the first step in the deductive method is supplied by Hypothesis.

By an Hypothesis is meant the assumption of some general law or, what comes to the same thing, the assumption of a cause to account for phenomena.

Hypothesis reverses the order of induction. In induc-

tion we first collect facts and from them infer some law :
in hypothesis we start by assuming a law and then look
for facts to support it.

If an hypothesis enables us to predict phenomena
with unfailing regularity, we hold it with nearly the same
certainty as if we had discovered a cause by observation
or experiment.

Conditions of a Valid Hypothesis.

(1) The cause assumed must be a *vera causa*, i. e.
something of a kind known to exist in nature.

(2) The operation we ascribe to it must not violate
any of the known laws of nature.

(3) It must be adequate to account for the phenomena.

(4) It must reconcile at least two different facts.

(5) It must admit of investigation, at least indirectly.

The hypothesis of the Ancients that the fires of Mount
Aetna were due to the giant Typhoeus being chained
beneath it—

> Alta iacet vasti super ora Typhoëos Aetne,
>   cuius anhelatis ignibus ardet humus
>
>   Ovid, *Fast.* IV. 491, 2.

fulfils none of these conditions : for

(1) Typhoeus is a fiction of the poets ;

(2) that he should breathe fire from his lungs con-
tradicts what we know of the laws of nature ;

(3) even granting that he did, could he go on
breathing fire age after age ?

(4) This would account only for one volcano.

(5) It defies investigation.

Contrast with this the hypothesis of a central fire, which not only accounts for volcanoes, but also for hot springs and the heat of mines and is consistent with what we know of the nature of the heavenly bodies.

For an hypothesis to be accepted, it ought not only to account for all the facts, but to be the only hypothesis that does account for them. E. g. In a criminal trial it is not sufficient to show that the prisoner had a motive and an opportunity for committing the crime, if it can be shown that others had also.

§ 198. *Crucial Instance.*

A Crucial Instance is some observation or experiment which decides a point under dispute. It is so called from *crux* in the sense of a sign-post, as pointing the way to truth, where there are conflicting hypotheses. A bacteriologist affords a crucial instance in his own person, when he dines off *bacilli*.

§ 199. *Analogy.*

I. Analogy in the strict sense, as used by Aristotle, means an equality of ratios (ἰσότης λόγων). We are said to argue from analogy, when, having laid down that—

$$\text{As } a : b : : c : d$$

we infer that what holds true of the relation between *a* and *b* will hold true also of the relation between *c* and *d*.

> E. g. As a child is to its parent, so is a colony to the mother-country.

Here we might argue that, since a child ought to obey its parent, a colony ought to obey the mother-country, or conversely, that since a parent ought to support the child, the mother-country ought to support the colony.

In either case we should be arguing from analogy in the ancient sense of the term.

Instances of analogy—
    As health : the body :: virtue : the soul.
    As old age : life :: evening : day.
    As the adjective : the substantive :: the adverb : the verb.

Argument from analogy—
    As flame : fuel :: light : flame.
    ∴ As the flame dies out with the fuel, so does the light with the flame.

II. In the loose modern sense Analogy means merely a high degree of resemblance between two things.

We are said to argue from analogy when, having found that A and B resemble each other in several points, we infer that they will resemble each other in some one point more. Analogy in this sense is the same thing as Example.

The value of such an argument must depend—

(1) On the importance of the ascertained points of resemblance.

(2) On the ratio which these bear to the ascertained points of difference.

(3) On the ratio which the ascertained points of resemblance and difference bear to the whole number of possible points of resemblance or difference.

Instance.   High degree of resemblance between the
Earth and Mars.

∴ Mars is inhabited.

If the points of difference outweigh those of resem-
blance, the force of the argument is turned the other way.

Instance.   The Moon differs from the Earth in impor-
tant points, e. g. in not having an atmosphere.

∴ The Moon is not inhabited.

In estimating the points both of resemblance and
difference care must be taken to see that they are inde-
pendent of one another. If there were four points of
resemblance, of which three were traceable as effects to
the remaining one, we should in that case really be
reduced to one point of resemblance.

Analogy is induction from a single instance, in which
the absence of number is compensated for by the high
degree of resemblance.

§ 200.   *Tendency.*

Nature's laws are never frustrated, but they are often
counteracted.   A body in the air has a tendency to fall to
the earth, but this effect does not take place so long as it
is supported.   A *tendency* may be defined as a potential
effect.

Explanation.

§ 201. To explain a thing is simply to show that it does
not stand alone, but is like other things.

We explain a fact when we refer it to a law, i. e. when
we show it to be a case of some wider fact.

We explain a law by referring it to laws more general than itself.

There are according to Mill (III. 12) three modes of explaining laws of causation—

> (1) By resolving the law of a complex effect into the laws of the separate causes, together with the fact of their coexistence.

E. g. The law of planetary motion is explained when it is shown to result from the combination of a centrifugal and a centripetal force.

> (2) By showing that what was supposed to be the immediate cause of an effect was only a remote one.

E. g. The action of the nerves is shown to intervene between contact with an object and sensation.

> (3) By showing a given law to be a case of some wider law.

E. g. The tendency of the moon towards the earth was shown to be a case of gravitation.

§ 202. *Different kinds of Laws.*

Sometimes any uniformity among physical phenomena is called a law of nature. But in a special sense—The Laws of Nature are the widest uniformities which can be arrived at, e. g. the law of gravitation, the laws of motion.

Derivative Laws, or Laws of Phenomena, are uniformities, which can be deduced from the laws of nature, e. g. the law of planetary motion.

Empirical Laws are special uniformities which are not deducible from any higher laws.

> e. g. A drake has a curly feather in its tail.
>
> Quinine is a cure for ague.
>
> Scarlet flowers have no scent.

CHAPTER XII.

Of Deductive Inferences.

§ 203. A DEDUCTIVE Inference is either immediate or mediate.

A Mediate Inference is so called because a middle term is employed.

An Immediate Inference is so called because no middle term is employed.

N. B.—A Mediate Inference is the same thing as a Syllogism.

An immediate inference is the comparison of two propositions directly.

A mediate inference is the comparison of two propositions by means of a third (namely, of the conclusion with the major premiss by means of the minor).

It may also be said that a mediate inference is the comparison of two terms by means of a third or middle term (namely, of the minor with the major by means of the middle).

In that sense of the term 'inference' in which it is confined to the consequent it may be said that—

An immediate inference is one derived from a single proposition.

A mediate inference is one derived from two propositions conjointly.

Where there appear to be more than two propositions in the antecedent of a mediate inference, the reasoning may always be resolved into more than one syllogism. Hence we may say that—

In an immediate inference the antecedent consists of one proposition.

In a mediate inference the antecedent consists of two propositions.

In an inductive inference the antecedent consists of one or more propositions.

In the case of the syllogism the two propositions which form the antecedent are called the Premisses; and the same term is sometimes employed for the propositions which make up the antecedent in an inductive inference. In all cases the consequent is also known as the Conclusion.

Immediate Inference.

§ 204. An immediate inference is either Simple or Compound.

A compound immediate inference is a combination of two or more simple immediate inferences.

Simple Immediate Inference.

§ 205. Of simple immediate inferences there are three kinds—

(1) by Opposition,
(2) by Conversion,
(3) by Obversion.

As it is impossible to get out of a proposition anything but what is contained in it, the consequent in an immediate inference must always be the antecedent or part of the antecedent under a disguised form.

In immediate inference by opposition there is a change of quantity or quality or both; in most cases also a change of material quality, i. e. a transition from truth to falsehood or the reverse.

In immediate inference by conversion there is a change in the position of the terms.

In immediate inference by obversion there is a change of quality both in the proposition itself and in its predicate.

If the proposition is changed to a further extent than is indicated by one of the three ways above mentioned, the immediate inference is compound.

CHAPTER XIII.

Of Immediate Inference by Opposition.

§ 206. BEFORE dealing with immediate inference by opposition we must say something about opposition itself, which is a relation between propositions.

Opposition.

Two propositions are said to be opposed when they are the same in matter but differ in form. It is plain that they must be the same in matter. There can be no opposition of any kind between two propositions which do not bear on the same point, e. g. 'Christmas is coming,' and 'No fishes talk.'

The propositions then which are opposed to one another must always have the same terms.

Now given any term S for a subject and any term P for a predicate, we can make up the four propositions A, E, I, O by taking account of quantity and quality—

All S is P.	(A)
No S is P.	(E)
Some S is P.	(I)
Some S is not P.	(O)

The relations between these four kinds of proposition may be indicated thus—
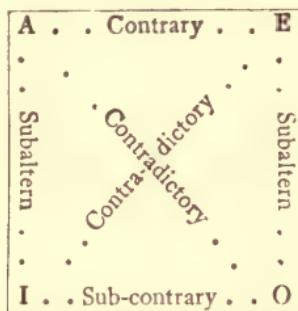
Contrary Opposition is the relation between two universal propositions which differ in quality.

Sub-contrary Opposition is the relation between two particular propositions which differ in quality.

Subaltern Opposition is the relation between two propositions which differ only in quantity.

Contradictory Opposition is the relation between two propositions which differ both in quantity and in quality.

Hence contradictory opposition is logically the strongest form. In the other kind of opposition there is a difference in quantity or quality, but not in both.

When the truth of one proposition is compatible with the truth of another, as in the case of subalterns and sub-contraries, there is no *opposition* in the ordinary meaning of the word between them, but the term is extended for convenience so as to cover any relation

between propositions based on a difference of quantity or quality or both.

Immediate Inference by Opposition.

§ 207. The problem of opposition regarded as a form of immediate inference is this—

Given the truth or falsehood of any one of the four propositions, A, E, I, O, what can be inferred with regard to the truth or falsehood of the others in the same matter?

§ 208. In order to solve this problem, we lay down the following

Laws of Opposition.

Contraries may both be false, but cannot both be true. Hence if one be true, the other is false, but not vice versâ.

Sub-contraries may both be true, but cannot both be false. Hence if one be false, the other is true, but not vice versâ.

Subalterns may both be true and may both be false.

But more particularly—

If the universal be true, the particular is true, but not vice versâ;

if the particular be false, the universal is false, but not vice versâ.

Contradictories cannot both be true and cannot both be false. Hence if one be true, the other is false, and vice versâ.

Application of the Laws.

§ 209. By applying these laws of opposition to the four kinds of proposition we obtain the following results—

If A be true, E is false,   I is true,   O is false.
If A be false, E is unknown, I is unknown, O is true.
If E be true, A is false,   O is true,   I is false.
If E be false, A is unknown, O is unknown, I is true.
If I be true, A is unknown, O is unknown, E is false.
If I be false, A is false,   O is true,   E is true.
If O be true, E is unknown, I is unknown, A is false.
If O be false, E is false,   I is true,   A is true.

Subalternation.

§ 210. Immediate inference by subaltern opposition is also known as Subalternation. The universal proposition is sometimes called the Subalternant and the particular the Subalternate.

§ 211. There is a peculiarity about immediate inference by opposition which should not be left unnoticed. In other forms of inference we argue from the truth of the antecedent to the truth of the consequent; in this we may argue also from the truth of the antecedent to the falsity of the consequent or from the falsity of the antecedent to the truth of the consequent or from the falsity of the antecedent to the falsity of the consequent. Now it is only in subalternation, when we start from the subalternant, that we are arguing from the truth of the

antecedent to the truth of the consequent. Nevertheless all the various forms of immediate inference by opposition fall under our definition of inferring as 'the passage of the mind from one or more judgements to another.' From the judgement 'this proposition is true' we pass to the judgement 'that proposition is false' and so on.

§ 212. A plausible objection has been urged against the doctrine that sub-contraries cannot both be false. It is this. I does not exclude A and O does not exclude E. This being so, it has been maintained that I and O may both be false, since they are possibly equivalent to A and E. But those who reason thus[1] are trying to get out of a proposition something more than there is in it. I and O make a statement about a part and are silent as to the whole. Though they do not exclude the universal as a matter of fact, they exclude it as a matter of argument. We are going beyond the evidence, if we assume the universal to be true. Moreover though it may as a matter of fact be true in one case, it cannot be true in both. It is a good instance of the fallacy of composition (§ 423) to argue thus—

> If I is true, A may be true; and if O is true, E may be true.

∴ If I and O are both true, A and E will be both true. A and E being contraries can never be true together.

§ 213. We have seen already that contradictory oppo-

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[1] As I did myself in my *Deductive Logic*; but my error was pointed out to me first by Mr. W. E. Johnson in *Mind* and afterwards by Mr. Keynes, to both of whom I am much obliged.

sites differ from one another in more respects than any other kind.

We may now add that they are incompatible both as to truth and falsehood, whereas the others are incompatible only as to one or the other. Lastly the contradictory is the proper refutation (*elenchus*) of an opponent's position. If we try to establish the contrary, we are liable to be refuted in turn. Let the opponent's position be 'All Red Indians are bad.' If we can prove that 'Some Red Indians are not bad,' we have refuted him; whereas if we try to prove that 'No Red Indians are bad,' we may be refuted in turn. The phrase 'diametrically opposed to one another' may have crept into common language from its use in 'the square of opposition[1].'

§ 214. The thing that is essential to contradiction is the double incompatibility as to truth and falsehood. This is best seen from

The Opposition of Singulars.

A singular proposition of which the subject is a concrete term, as dealing with a primary substance, may be regarded as the elementary type of proposition. It is individual, but not particular. Consequently there can be no distinction between contrary and contradictory or between subalternant and subalternate. Hence opposi-

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[1] Simplicius Comm. in Ench. cap. xlviii has ἐκ διαμέτρου τούτων and again τὰ κατὰ διάμετρον ἀλλήλων. Eus. Pr. Ev. xv 32 § 6 ἐναντίοι κατὰ διάμετρον. The Scholiast on Lucian, Cataplus 14, explains that the metaphor is derived from a circle.

tion is in this case reduced to one kind, namely, contra-
diction. The contradictory of 'Francis was the author
of the Letters of Junius' is 'Francis was not the author
of the Letters of Junius.'

A kind of generality may be imparted to a singular
proposition by expressing it in the form 'This S is
always P.' We may then give

<blockquote>
its contrary as   'This S is never P,'

its subaltern as   'This S is sometimes P,'

its contradictory as 'This S is sometimes not P.'
</blockquote>

§ 215. Immediate inference by opposition may be
defined as an immediate inference based on the relation
to one another of two propositions which are the same in
matter but different in form, i.e. in quantity or quality or
both.

CHAPTER XIV.

Of Immediate Inference by Conversion.

§ 216. A proposition is said to be converted when its subject and predicate are transposed. When this is done in such a way that the second proposition follows from the first, we have the logical converse of the original proposition.

Immediate Inference by Conversion.

§ 217. In this form of immediate inference
the antecedent is known as the Convertend,
the consequent is known as the Converse.

Two Rules for Conversion.

(1) No term must be distributed in the converse which was not distributed in the convertend.

(2) There must be no change of quality.

The former of these rules is founded in the nature of things : we cannot argue safely from the part to the whole.

The latter is conventional : we may draw a valid inference in spite of it, but it will involve something more than conversion.

Two kinds of Conversion.

§ 218. (1) Simple,   (2) By limitation or *per accidens*.
In simple conversion there is no change of quantity.

In conversion by limitation the quantity of the original proposition is reduced.

E and I can be converted simply,

A and E can be converted by limitation,

O cannot be converted.

A proposition, it will be seen, admits of being simply converted when the quantity of the subject is the same as that of the predicate: otherwise not.

E.　　No $\bar{S}$ is $\bar{P}$.
　　∴ No $\bar{P}$ is $\bar{S}$.

I.　　Some $\breve{S}$ is P.
　　∴ Some $\breve{P}$ is $\breve{S}$.

A.　　All $\bar{S}$ is $\breve{P}$.
　　∴ Some $\breve{P}$ is $\breve{S}$.

§ 219. *Reason why* A *must be converted by limitation.*

A, being affirmative, does not distribute its predicate. Therefore when the predicate is made the subject, the proposition becomes particular.

All butchers are mĕn.
∴ Some mĕn are butchers.

§ 220. *Reason why* O *cannot be converted.*

O, being particular, does not distribute its subject. But when the subject is made the predicate, it will have to be distributed, if the proposition is negative. Therefore either there must be a change of quality or there will be

a term distributed in the converse which was not distributed in the convertend.

> Some animals are not dogs.
> ∴ Some dogs are not animals.

§ 221. The simple converse of an A proposition may often be true along with it, but its truth does not follow from that of the other and has to be established on separate evidence, e. g.

> All equilateral triangles are equiangular,
> and All equiangular triangles are equilateral.

§ 222. On the theory of the quantified predicate every proposition admits of simple conversion, being an equation between its terms. This is the case in some natural forms of expression, where the proposition is singular, e. g.

> (υ) Virtue is the condition of happiness.
> (a) Virtue is a condition of happiness.
> (υ) The square of three is nine.

§ 223. When the predicate is an attributive, it has to be changed into a subject-term in conversion, e. g.

> Some mistakes are criminal.
> ∴ Some crimes are mistakes.
> No lie ever thrives in the long run.
> ∴ Nothing which ever thrives in the long run is a lie.

§ 224. Immediate inference by conversion may be defined as an immediate inference based on the transposition of the subject and predicate in a proposition.

CHAPTER XV.

Of Immediate Inference by Obversion.

§ 225. To obvert a proposition is to change its quality without changing its meaning. In other words it is expressing negatively what was expressed affirmatively or vice versâ.

If we only changed the quality of a proposition, we should get its contrary or sub-contrary, as the case might be, and should therefore seriously affect its meaning. But this change is set right by a further change in the quality of the predicate, for which we substitute the contradictory term.

§ 226. Thus obversion rests on the principle that two negatives make an affirmative. It is an exemplification of the Law of Excluded Middle, namely, that one or other of a pair of contradictory terms must be applicable to a given subject, so that, when one may be predicated affirmatively, the other may be predicated negatively, and vice versâ.

§ 227. Strictly speaking it should always be the contradictory of the original predicate that is employed in obversion, but in practice we may substitute a privative for a negative term. The inference will still hold, since

a proposition is always made with reference to some given sphere of thought, within which positive and privative are as mutually incompatible as positive and negative. The meaning of this remark will be evident from the examples.

(A)   All S is P.   All just acts are expedient.

(E)   ∴ No S is not-P.   ∴ No just acts are inexpe-
                                    dient.

(E)   No S is P.   No display of passion is
                                    politic.

(A)   ∴ All S is not-P.   ∴ Every display of passion
                                    is impolitic.

(I)   Some S is P.   Some philosophers have
                                    been slaves.

(O)   ∴ Some S is not not-P.   ∴ Some philosophers have
                                    not been free.

(O)   Some S is not P.   Some tyrants have not
                                    been unprosperous.

(I)   ∴ Some S is not-P.   ∴ Some tyrants have been
                                    prosperous.

§ 228. Obversion is also known as Permutation and as Immediate Inference by Privative Conception.

It may be defined as an immediate inference in which there is a change of quality both in the proposition itself and in its predicate, but no transposition of terms.

CHAPTER XVI.

Of Compound Immediate Inferences.

§ 229. Compound forms of immediate inference are obtained by combining two or more simple forms. If we both obvert and convert the same proposition we have

Conversion by Negation.

§ 230. Conversion by Negation is a compound immediate inference, in which the terms are transposed and there is a change of quality both in the proposition itself and in one of the terms.

It may assume two forms.

In one the consequent has the contradictory of the original predicate for its subject and the original subject for its predicate.

In the other the consequent has the original predicate for its subject and the contradictory of the original subject for its predicate.

(i)

A. All S is P.	E. No S is P.	O. Some S is not P.
E. ∴ No not-P is S.	I. ∴ Some not-P is S.	I. ∴ Some not-P is S.
All lawyers are honest.	No just man is partial.	Some statesmen are not practical.
∴ No dishonest men are lawyers.	∴ Some impartial men are just.	∴ Some unpractical men are statesmen.

When the quality of the original predicate is to be changed we must first obvert and then convert.

In the case of E limitation has to be employed, so that its converse by negation coincides with that of O. I can yield no result, because when obverted it becomes O, which cannot be converted.

<div style="text-align:center">(ii)</div>

A. All S is P.	No S is P.	I. Some S is P.
O. ∴ Some P is not not-S.	A. ∴ All P is not-S.	O. ∴ Some P is not not-S.
All safe ways are certain.	No selfish acts are praiseworthy.	Some critics are kind.
∴ Some certain ways are not unsafe.	∴ All praiseworthy acts are unselfish.	∴ Some kind people are not uncritical.

When the quality of the original subject is to be changed, we must first convert and then obvert.

In the case of A limitation has to be employed, so that its converse by negation coincides with that of I. O can yield no result, because it cannot be converted.

§ 231. If we carry the first form one step further by obverting the result, we have

Conversion by Contraposition.

Conversion by Contraposition is a compound immediate inference in which the terms are transposed and their quality changed.

A. All S is P.	E. No S is P.	O. Some S is not P.
A. ∴ All not-P is not-S.	O. Some not-P is not not-S.	O. Some not-P is not not-S.
		not-S.

All solid substances are material.	Nothing familiar is striking.	Some prejudiced persons are not dishonest.
∴ All immaterial substances are non-solid.	∴ Some unstriking things are not unfamiliar.	∴ Some honest persons are not unprejudiced.

N.B.—If one of the terms be already privative or negative, the change of quality will make it positive, as in the concrete example of O.

The above resolve themselves into three steps of simple immediate inference in this order—

(1) Obversion,
(2) Conversion,
(3) Obversion.

In the case of E limitation has to be employed, so that its contrapositive coincides with that of O.

§ 232. U admits of an inference by contraposition without conversion.

All S is all P.	All equilateral triangles are equiangular.
∴ All **not**-S is not-P.	∴ All non-equilateral triangles are non-equiangular.

The principle upon which this kind of inference rests is that when two terms are coextensive, whatever is excluded from the one is excluded also from the other.

§ 233. If we carry the second form one step further by converting the result we have what Mr. Keynes calls

Inversion.

Inversion is a compound immediate inference in which

there is a change of quality both in the proposition itself and in its subject, but no transposition of terms.

This form of inference applies to E and A.

E.   No S is P.   No Jews are Aryans.

I.   ∴ Some not-S is P.   ∴ Some other than Jews are
                                             Aryans.

The above resolves itself into three steps of simple immediate inference in this order—

    (1) Conversion.

    (2) Obversion.

    (3) Conversion.

A.   All S is P.   All Jews are Semitic.

O.   ∴ Some not-S is not P.   ∴ Some other than Jews
                                     are not Semitic.

The above involves no less than five steps of obversion-conversion.

§ 234. We have seen that the inverse of E is I, and that it is reached by three steps of conversion-obversion. If we carry on the same process one step further and obvert the inverse, we reach O—

               Some not-S is not not-P.

O is a terminus. When we have arrived at it, we can go no further either by conversion-obversion or by obversion-conversion: for we cannot convert O and, if we obvert it, we are only retracing our steps. Even then however there are still some side-branches on which we can travel. This will be seen, if we take each of the four propositions and try to draw therefrom all possible inferences.

A.

All S is P.	Original.
No S is not-P.	Obverse.
No not-P is S.	Converse by Negation (i).
All not-P is not-S.	Contrapositive.
Some not-S is not-P.	Converse of the Contrapositive (=Obverse of the Inverse).

(o) Some not-S is not P.	Inverse.
Some P is S.	Converse.
(o) Some P is not not-S.	Converse by Negation (ii).
Some S is P.	Subaltern.
Some S is not not-P.	Subaltern of the Obverse.
Some not-P is not-S.	Subaltern of the Contrapositive.
Some not-P is not S.	Subaltern of the Converse by Negation (i).

E.

No S is P.	Original.
All S is not-P.	Obverse.
Some not-P is S.	Converse by Negation (i).
(o) Some not-P is not not-S.	Contrapositive.

No P is S.	Converse.
All P is not-S.	Converse by Negation (ii).
Some not-S is P.	Inverse.
(o) Some not-S is not not-P.	Obverse of the Inverse.

Some S is not P.	Subaltern.
Some S is not-P.	Subaltern of the Obverse.
Some P is not S.	Subaltern of the Converse.
Some P is not-S.	Subaltern of the Converse by Negation (ii).

I.

Some S is P.	Original.
(o) Some S is not not-P.	Obverse.
Some P is S.	Converse.
(o) Some P is not not-S.	Obverse of the Converse.

O.

Some S is not P.	Original.
Some S is not-P.	Obverse.
Some not-P is S.	Converse by Negation (i).
(o) Some not-P is not not-S.	Contrapositive.

§ 235. The foregoing lists show the wealth of implication that is contained in a universal proposition. When we assert A or E, we are committing ourselves to a dozen different statements in each case. If any one of the eleven be false, the original proposition cannot be true, so that we have ample means of putting it to the test. We cannot however argue that if any one of the eleven be true, the original proposition is true. This is the case only when there is no loss of quantity. There are eleven propositions which can be derived from A, but only from three of them can A be recovered. The same holds true

of E. On the other hand I and O can be recovered from any of the three propositions which are derived from them.

§ 236. There is unfortunately much difference of nomenclature with regard to compound immediate inferences, so that it is impossible to be in accord with everyone. But before quitting the subject it may be well to explain the principle upon which we have gone.

Negation properly means the substitution of a negative for a positive term : but it may be taken to cover a change of quality in either direction. This change of quality may affect either the subject or the predicate. When it affects both, it is convenient to have another name for it and to call it Contraposition. Thus Contraposition is the same thing as Double Negation.

Hence Conversion by Negation is distinguished from Conversion by Contraposition by the fact that in the former process only one of the terms has its quality changed, whereas in the latter both have. Moreover in Conversion by Negation the quality of the proposition itself is changed, whereas in Conversion by Contraposition it is not.

In Inversion also the quality of one of the terms (namely, of the subject) is changed, but as there is no transposition of terms, it need not be confused with Conversion by Negation.

Inversion is to the subject of a proposition what Obversion is to the predicate.

All the four processes of Obversion, Conversion by

Negation, Conversion by Contraposition, and Inversion involve a change of quality in the terms.

If the quality of the subject is changed, and the terms are not transposed, we have Inversion.

If the quality of the predicate is changed, and the terms are not transposed, we have Obversion.

If the quality of the subject only is changed, and the terms are transposed, we have Conversion by Negation (ii).

If the quality of the predicate only is changed, and the terms are transposed, we have Conversion by Negation (i).

If the quality of both subject and predicate is changed, and the terms are transposed, we have Conversion by Contraposition.

CHAPTER XVII.

Of Other Forms of Immediate Inference.

§ 237. Having treated of the main forms of immediate inference, whether simple or compound, we will now close this subject with a brief allusion to some other forms which have been recognised by logicians.

§ 238. Every statement of a relation may furnish us with an immediate inference in which the same fact is presented from the opposite side. Thus from ' John hit James' we infer ' James was hit by John;' from ' Dick is the grandson of Tom' we infer ' Tom is the grandfather of Dick;' from ' Bicester is north-east of Oxford' we infer ' Oxford is south-west of Bicester;' from ' So and so visited the Academy the day after he arrived in London' we infer ' So and so arrived in London the day before he visited the Academy;' from ' A is greater than B' we infer ' B is less than A;' and so on without limit. Such inferences as these are material, not formal. No law can be laid down for them except the universal postulate, that

> ' Whatever is true in one form of words is true in every other form of words which conveys the same meaning.'

§ 239. There is a sort of inference which goes under the title of Immediate Inference by Added Determinants, in which from some proposition already made another is inferred, in which the same attribute is attached both to the subject and the predicate, e. g.

> A horse is a quadruped.
> ∴ A white horse is a white quadruped.

Such inferences are very deceptive. The attributes added must be definite qualities, like whiteness, and must in no way involve a comparison. From 'A horse is a quadruped' it may seem at first sight to follow that 'A swift horse is a swift quadruped.' But we need not go far to discover how little formal validity there is about such an inference. From 'A horse is a quadruped' it by no means follows that 'A slow horse is a slow quadruped;' for even a slow horse is swift compared with most quadrupeds. All that really follows here is that 'A slow horse is a quadruped which is slow for a horse.' Similarly, from 'A bushman is a man' it does not follow that 'A tall bushman is a tall man,' but only that 'A tall bushman is a man who is tall for a bushman;' and so on generally.

§ 240. Very similar to the preceding is the process known as Immediate Inference by Complex Conception, e. g.

> A horse is a quadruped.
> ∴ The head of a horse is the head of a quadruped.

This inference, like that by added determinants, from which it differs in name rather than in nature, may be

explained on the principle of Substitution. Starting from
the identical proposition, 'The head of a quadruped is
the head of a quadruped,' and being given that 'A horse
is a quadruped,' so that whatever is true of 'quadruped'
generally we know to be true of 'horse,' we are entitled to
substitute the narrower for the wider term, and in this
manner we arrive at the proposition,

The head of a horse is the head of a quadruped.

Such an inference is valid enough, if the same caution
be observed as in the case of added determinants, that
is, if no difference be allowed to intervene in the rela-
tion of the fresh conception to the generic and the specific
terms.

CHAPTER XVIII.

Of Immediate Inference as applied to Complex Propositions.

§ 241. Before entering on this subject it will be well to show first that the fourfold division (§ 52) applies as much to complex as to simple propositions.

§ 242. Conjunctive Propositions may assume any of the four forms, A, E, I, O, as follows—

A. If A is B, C is always D.

E. If A is B, C is never D.

I. If A is B, C is sometimes D.

O. If A is B, C is sometimes not D[1].

---

[1] The reader should be warned at once against the ambiguity of this last form of expression. It may mean

(1) If A is B, it is sometimes not true that C is D.   O.

(2) If A is B, it is sometimes true that C is not D.   I.

That is to say, it fails to distinguish between the proposition itself and its obverse. Whenever this becomes of any importance the full forms just given ought to be employed. The same purpose however may be served by putting a hyphen between the 'sometimes' and the 'not' when the proposition is to be read as O, thus—

If A is B, C is sometimes-not D.   O.

These admit of being read in the form of simple propositions, thus—

If A is B, C is always D	=All AB is CD.	A.
If A is B, C is never D	=No AB is CD.	E.
If A is B, C is sometimes D	=Some AB is CD.	I.
If A is B, C is sometimes not D	=Some AB is not CD.	O.

If kings are ambitious, their subjects always suffer.

=All cases of ambitious kings are cases of subjects suffering. A.

If the wind is in the south, the river never freezes.

=No cases of wind in the south are cases of the river freezing. E.

If a man plays recklessly, the luck sometimes goes against him.

=Some cases of reckless playing are cases of luck going against one. I.

If a book has merit, the public sometimes do not buy it.

=Some cases of books having merit are not cases of the public buying. O.

§ 243. The difference between the conjunctive and the disjunctive proposition lies, as we have seen, in this—that in the conjunctive the truth, in the disjunctive the falsity, of the antecedent implies the truth of the consequent. The disjunctive proposition therefore may be read as a conjunctive, consequently as a simple proposition with a negative term for its subject.

Either A is B or C is D.

=If A is not B, C is D.

=All not-AB is CD.

Hence what has just been said of the conjunctive proposition becomes applicable to the disjunctive, which may be constructed in all four forms, but is commonly treated as if it were always A.

Either A is B or C is always D	= All not-AB is CD.	A.
Either A is B or C is never D	= No not-AB is CD.	E.
Either A is B or C is sometimes D	= Some not-AB is CD.	I.
Either A is B or C is sometimes not D	= Some not-AB is not CD.	O.

Either men are immortal or their hopes are always deceived.

= All cases of men being mortal are cases of their hopes being deceived.   A.

Either the sky is clear or there is never dew.

= No cases of the sky being cloudy are cases of dew.   E.

Either children tell the truth or else sometimes they are frightened.

= Some cases of children not telling the truth are cases of their being frightened.   I.

Either he wins easily or else sometimes he does not attend to the game.

= Some cases of his not winning easily are cases in which he does not attend to the game.   O.

§ 244. It must be understood that in the complex proposition of which we are speaking both antecedent and consequent are indefinite and either affirmative or treated as such. The signs of quantity and quality, 'always,' 'never,' &c., though they may appear in the consequent, do not refer to that proposition itself, but to its connexion with the antecedent. We will not pursue

the complexities that would arise, if we were to quantify
and qualify the antecedent and consequent themselves as
well as the whole proposition of which they are the terms.
It will be sufficient to indicate these by a single instance—

If some A is B, it is always true that some C is not D.

If some instances of an experiment are failures, it is
    always true that some of its conditions are not fulfilled.

Here the whole proposition is A, the antecedent I, the
consequent O.

§ 245. The disjunctive proposition is often treated as
though it conveyed two statements in one breath.   Yet it
ought not, any more than the E proposition, to be regarded
as conveying both with equal directness.   The proposition
'No S is P' is not considered to assert directly, but only
implicitly, that 'No P is S.'   In the same way the form
'Either A is B or C is D' ought to be interpreted as
meaning directly no more than this, 'If A is not B, C is D.'
It asserts indeed by implication also, that 'If C is not D,
A is B.'   But this is an immediate inference from the
proposition, not the proposition itself.   When General
Mercier declares 'Some one is guilty; it is either Dreyfus
or I,' he wishes us to read the proposition thus—

'If I am not guilty, Dreyfus must be guilty.'

The alternative rendering

'If Dreyfus is not guilty, I must be guilty,' is only an
inconvenient corollary.   The inference indeed is so inevit-
able that it seems indistinguishable from the former
proposition: but, since the two members of a complex
proposition play the part of subject and predicate, to say

that the two statements are identical would be tantamount to asserting that the same proposition can have two subjects and two predicates. At the same time we may grant that, when a proposition is expressed in disjunctive form, it is indifferent which term we select as subject.

§ 246. We are now in a position to see how immediate inference may be applied to complex propositions.

Opposition.

§ 247. The relation of propositions in the same matter to one another may be seen as clearly in the complex as in the simple form. Let us take as an instance a disjunctive A proposition.

A. Either A is B or C is always D.
E. Either A is B or C is never D.   Contrary.
I. Either A is B or C is sometimes D.   Subaltern.
O. Either A is B or C is sometimes not D. Contradictory.

Conversion.

§ 248. Conjunctive propositions may be converted as easily as simple ones.

A. If A is B, C is always D.
  ∴ If C is D, A is sometimes B.

E. If A is B, C is never D.
  ∴ If C is D, A is never B.

I. If A is B, C is sometimes D.
  ∴ If C is D, A is sometimes B.

§ 249. That the converse of A is the proposition given

above may be seen by our first reducing the complex proposition to simple form, then converting it, and finally throwing back the result into complex form.

> A.   If A is B, C is always D.
> = All AB is CD.
> ∴ Some CD is AB.
> = If C is D, A is sometimes B.

§ 250. The last proposition must not be misunderstood as though it contained an assertion of fact. The meaning might be better conveyed by the form

> If C is D, it may be that A is B.

A concrete instance will render this point clearer.

'If kings are ambitious, their subjects always suffer' may be converted into

> 'If subjects suffer, it may be that their kings are ambitious,'

i. e. among the possible causes of suffering on the part of subjects is to be found the ambition of their rulers, even if every actual case should be rightly referred to some other cause. It is in this sense only that the inference is a necessary one. But then this is the only sense with which we are concerned.

To judge of conformity to fact is no part of the province of deduction. From 'All AB is CD' it follows that 'Some CD is AB' with the same necessity as that with which 'Some P is S' follows from 'Some S is P.' In the latter case also neither proposition may conform to fact.

From 'All centaurs are animals' it follows necessarily that 'Some animals are centaurs,' but as a matter of fact

this is not the case. The consequent, if rightly drawn, will always be *as* true as the antecedent.

All Centaurs (if there be such creatures) are animals.

∴ Some animals are Centaurs (if there be such creatures).

§ 251. The O proposition can of course no more be converted in the complex than in the simple form. From the proposition

'If a man runs a race, he sometimes does not win it,' it certainly does not follow that

'If a man wins a race, he sometimes does not run it.'

§ 252. Disjunctive propositions cannot be converted without losing their disjunctive form, the reason being that the consequent is affirmative, so that when it is made the antecedent, the proposition assumes the conjunctive form.

A.   Either A is B or C is always D.

     ∴ If C is D, it may be that A is not B.

E.   Either A is B or C is never D.

     ∴ If C is D, it is never true that A is not B.

I.   Either A is B or C is sometimes D.

     ∴ If C is D, it may be that A is not B.

Obversion.

§ 253. A.   If A is B, it is always true that C is D.

     ∴ If A is B, it is never true that C is not D. E.

     If a mother loves her children, she is always kind to them.

     ∴ If a mother loves her children, she is never unkind to them.

E.   If A is B, it is never true that C is D.

∴ If A is B, it is always true that C is not D.   A.

If a man tells lies, his friends never trust him.

∴ If a man tells lies, his friends always distrust him.

I.   If A is B, it is sometimes true that C is D.

∴ If A is B, it is sometimes not true that C is not D.   O.

If strangers are confident, savage dogs are sometimes friendly.

∴ If strangers are confident, savage dogs are sometimes not unfriendly.

O.   If A is B, it is sometimes not true that C is D.

∴ If A is B, it is sometimes true that C is not D.   I.

If a measure is good, its author is sometimes not popular.

∴ If a measure is good, its author is sometimes unpopular.

§ 254. The disjunctive proposition may be obverted as it stands without being reduced to the conjunctive form. It will be sufficient to indicate this by a single example.

Either A is B or C is D.

∴ Either A is B or C is not not-D.

Either a sinner must repent or he will be damned.

∴ Either a sinner must repent or he will not be saved.

Conversion by Negation.

§ 255. It will be expedient now to confine ourselves, at least in the abstract examples, to a form of expression which leaves no chance of the quantity and quality of the

whole proposition being confounded with that of the consequent.

The student must bear in mind that in conversion by negation there is always a change of quality in the proposition, that there is always a transposition of terms, and that the quality of one of the terms is always changed. Further he must bear in mind that the process admits of two forms—

> that in the first the original subject remains unchanged,

> that in the second the original predicate remains unchanged.

§ 256. We have first to apply this mode of inference to conjunctive propositions.

<div align="center">(i.)</div>

A.   If A is B, it is always true that C is D.

∴ If C is not D, it is never true that A is B.   E.

If a man is a smoker, he always drinks.

∴ If a man is a total abstainer, he never smokes.

E.   If A is B, it is never true that C is D.

∴ If C is not D, it is sometimes true that A is B.   I.

If the wind is high, rain never falls.

∴ If rain does not fall, the wind is sometimes high.

O.   If A is B, it is sometimes not true that C is D.

∴ If C is not D, it is sometimes true that A is B.   I.

If the ground is wet, there has sometimes not been rain.

∴ If there has not been rain, the ground is sometimes wet.

(ii.)

A.   If A is B, it is always true that C is D.

∴ If C is D, it is sometimes not true that A is not B. o.

If money is scarce, prices are always low.

∴ If prices are low, money is sometimes not abundant.

E.   If A is B, it is never true that C is D.

∴ If C is D, it is always true that A is not B.   A.

If a man merely does his duty, no one ever thanks him.

∴ If people thank a man, he has always done more than his duty.

I.   If A is B, it is sometimes true that C is D.

∴ If C is D, it is sometimes not true that C is not D. o.

If a statesman is patriotic, he sometimes changes his party.

∴ If a statesman changes his party, he is sometimes not unpatriotic.

§ 257. Next we have to apply the same mode of inference to disjunctive propositions.

(i.)

A.   Either A is B or it is always true that C is D.

∴ Either C is D or it is never true that A is not B. E.

Either miracles are possible or ancient historians are always untrustworthy.

∴ Either ancient historians are untrustworthy or miracles are not impossible.

E.   Either A is B or it is never true that C is D.

∴ Either C is D or it is sometimes true that A is
     not B.   1.

   Either he loses his temper or nothing is done to
   displease him.

∴ If something is not done to displease him, he
   sometimes does not lose his temper.

O.   Either A is B or it is sometimes not true that C
      is D.

∴ Either C is D or it is sometimes true that A is
   not B.   1.

   Either there is a future life or it is sometimes not
   true that justice prevails.

∴ Either justice prevails or it is sometimes true
   that there is not a future life.

In the above examples the quality of the original
subject is retained, while the predicate is changed into
its contradictory.   These facts are obscured to the eye
by the negative force which the word 'either' imparts to
the antecedent of a disjunctive proposition.

(ii.)

A.   Either A is B or it is always true that C is D.

∴ Either C is not D or it is sometimes not true
   that A is B.   0.

   Either there is more than one principle at work
   or evil is merely apparent.

∴ Either evil is not merely apparent or there may
   be no more than one principle at work.

A.   Either A is B or it is never true that C is D.

     ∴ Either C is not D or it is always true that A is B.   A.

       Either the rivers will run back to their sources or the country will not be abandoned.

     ∴ Either the country will not be abandoned or the rivers will run back to their sources.

I.   Either A is B or it is sometimes true that C is D.

     ∴ Either C is not D or it is sometimes not true that A is B.   o.

       Either things are very much amiss or godliness is sometimes gain.

     ∴ Either godliness is not gain or things are sometimes not very much amiss.

In the above examples the quality of the original predicate is retained, while the subject is changed into its contradictory. These facts are obscured to the eye by the same cause as before.

Whenever the converse by negation of a proposition seems difficult to grasp, let the reader mentally reduce it to the equivalent conjunctive, e. g.

     Either godliness is not gain or things are sometimes not very much amiss.

= If godliness is gain, things are sometimes not very much amiss.

Conversion by Contraposition.

§ 258. First as applied to conjunctive propositions.

A.   If A is B, it is always true that C is D.

     ∴ If C is not D, it is always true that A is not B.   A.

If a man is a smoker, he always drinks.

∴ If a man is a total abstainer, he is always a non-smoker.

E.   If A is B, it is never true that C is D.

∴ If C is not D, it is sometimes not true that A is not B. o.

If the wind is high, rain never falls.

∴ If rain does not fall, the wind is sometimes not low.

O.   If A is B, it is sometimes not true that C is D.

∴ If C is not D, it is sometimes not true that A is B. o.

If the ground is wet, there has sometimes not been rain.

∴ If there has not been rain, the ground is sometimes not unwet.

§ 259. Next as applied to disjunctive propositions.

A.   Either A is B or it is always true that C is D.

∴ Either C is D or it is always true that A is B. A.

Either miracles are possible or ancient historians are always untrustworthy.

∴ Either ancient historians are untrustworthy or miracles are always possible.

E.   Either A is B or it is never true that C is D.

∴ Either C is D or it is sometimes not true that A is B. o.

Either he loses his temper or nothing is done to displease him.

∴ Either something is done to displease him or he sometimes does not lose his temper.

o.   Either A is B or it is sometimes not true that C is D.

    ∴ Either C is D or it is sometimes not true that A is B. o. Either there is a future life or it is sometimes not true that justice prevails.

    ∴ Either justice prevails or it is sometimes not true that there is a future life.

§ 260. On comparing the above results with the converse by negation (i.) of each of the same propositions, A, E, O, the reader will see that they differ from the latter, as was to be expected, only in being obverted (§ 231). The validity of the inference may be tested, both here and in the case of conversion by negation, by reducing the disjunctive proposition to the conjunctive, and so to the simple form, then performing the required process as in simple propositions, and finally throwing the result, when so obtained, back through the conjunctive into the disjunctive form. We will show in this manner that the above is really the contrapositive of the O proposition.

o.   Either A is B or it is sometimes not true that C is D.
                (Disjunctive.)

  = If A is not B, it is sometimes not true that C is D.
                (Conjunctive.)

  = Some not-AB is not CD.   (Simple.)

  ∴ Some not-CD is not AB.   (Contrapositive.)

  = If C is not D, it is sometimes not true that A is B.
                (Conjunctive.)

  = Either C is D or it is sometimes not true that A is B.
                (Disjunctive.)

> Either there is a future life or it is sometimes not true
>> that justice prevails.

= If there be not a future life, is is sometimes not true
>> that justice prevails.

= Some cases of there not being a future life are not
>> cases of justice prevailing.

∴ Some cases of justice not prevailing are not cases of
>> there being a future life.

= If justice do not prevail, it is sometimes not true that
>> there is a future life.

= Either justice prevails or it is sometimes not true that
>> there is a future life.

Inversion.

§ 261. This form of inference is applicable only to E
and A propositions.

§ 262. We will take these two propositions first in the
conjunctive form.

E. If A is B, it is never true that C is D.

∴ If A is not B, it is sometimes true that C is D.   1.
> If a man merely does his duty, no one ever thanks him.

∴ If a man does more than his duty, people sometimes
>> thank him.

If the reader compares this with the converse by nega-
tion (ii.) of E, he will find that it differs from it only in
being converted.

A. If A is B, it is always true that C is D.

∴ If A is not B, it is sometimes not true that C is D.   o.
> If a man is a smoker, he always drinks.

∴ If a man is a non-smoker, he sometimes does not drink.

It would take five steps of obversion-conversion to reach this conclusion from A. Let us verify this assertion.

A.   If a man is a smoker, he always drinks.

E. ∴ If a man is a smoker, he is never a total abstainer.
                                        (Obversion.)

E. ∴ If a man is a total abstainer, he is never a smoker.
                                        (Conversion.)

A. ∴ If a man is a total abstainer, he is always a non-smoker.   (Obversion.)

I. ∴ If a man is a non-smoker, he is sometimes a total abstainer.   (Conversion.)

O. ∴ If a man is a non-smoker, he sometimes does not drink.   (Obversion.)

§ 263. We will next take E and A in the disjunctive form.

E.   Either A is B or it is never true that C is D.

  ∴ Either A is not B or it is sometimes true that C is D. I.

      Either the rivers will run back to their sources or the country will not be abandoned (=No not-AB is CD).

  ∴ Either the rivers will not run back to their sources or in some cases the country will be abandoned (= Some AB is CD).

This conclusion differs from the converse by negation (ii.) of E only in being converted. It is however so far from being self-evident that it may be well to work it out.

The inverse of E, it will be remembered, is reached by the following three steps—

        (1) Conversion,
        (2) Obversion,
        (3) Conversion.

    Either A is B or it is never true that C is D.

                                     (Disjunctive)

= If A is not B, it is never true that C is D.

                                     (Conjunctive)

= No not-A B is CD.   (Simple)

∴ No CD is not A B.   (Converse)

∴ All CD is A B.   (Obverse)

∴ Some A B is CD.   (Converse)

=   If A is B, it is sometimes true that C is D.

                                     (Conjunctive)

=   Either A is not B or it is sometimes true that C is D.   (Disjunctive)

A.   Either A is B or it is always true that C is D.

   ∴ Either A is not B or it is sometimes not true that C is D.

     Either miracles are possible or ancient historians are always untrustworthy.

   ∴ Either miracles are not possible or ancient historians are sometimes not untrustworthy.

This conclusion will be found to be the converse by negation (ii.), i.e. the obverted converse, of the contrapositive of the original.

CHAPTER XIX.

Of Mediate Inferences or Syllogisms.

§ 264. A syllogism is a combination of three propositions, in which the third follows from the two conjointly. The two propositions are called the Premisses, the third the Conclusion.

<table>
<tr><td></td><td colspan="2"><i>Middle Term.   Major Term.</i></td><td rowspan="2">Antecedent
or
Premisses.</td></tr>
<tr><td>Major Premiss.</td><td colspan="2">All mammals are warm-blooded.</td></tr>
<tr><td></td><td colspan="2"><i>Minor Term.   Middle Term.</i></td><td></td></tr>
<tr><td>Minor Premiss.</td><td colspan="2">All whales are mammals.</td><td></td></tr>
</table>

<table>
<tr><td colspan="2"><i>Minor Term.   Major Term.</i></td><td rowspan="2">Consequent
or
Conclusion.</td></tr>
<tr><td>∴</td><td>All whales are warm-blooded.</td></tr>
</table>

§ 265. As a syllogism consists of three propositions and a proposition consists of two terms, there are in one sense six terms in a syllogism. In another sense there are only three, since each term occurs twice over.

The middle term occurs twice in the premisses and does not appear in the conclusion.

Each of the other terms occurs once in a premiss and once in the conclusion.

The Major Term is the predicate of the conclusion.

The Minor Term is the subject of the conclusion.

The Middle Term is that by which the subject and predicate of the conclusion are brought into connexion with one another.

The major and minor term are called the Extremes as opposed to the Mean or middle term.

§ 266. The Major Premiss is that which contains the major term.

The Minor Premiss is that which contains the minor term.

The major premiss is generally put first, but this is not necessary and it is doubtful whether it is desirable. 'Whales—mammals—warm-blooded' is the order of thought, which is represented by writing the premisses in the reverse order.

> All whales are mammals.
> All mammals are warm-blooded.
> ∴ All whales are warm-blooded.

The reason why the names 'major,' 'middle,' and 'minor' terms were originally employed is that in a syllogism such as the above, which was regarded as the perfect type of syllogism, these names represent the relative quantity in extension of the three terms.
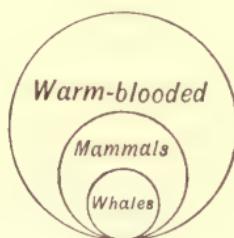
It must be noticed however that, though the major term cannot be of less extent than the middle nor the middle than the minor, there is nothing to prevent any two, or all three, of them from being coextensive. The latter is the case in Traduction.

If the minor premiss were particular instead of universal, the names would not apply. We should then have either a case of the intersection of two classes, from which it could not be told which of them was the larger, or the minor term would actually be larger than the middle, standing to it in the relation of genus to species, as in the following syllogism—

> All Negroes have woolly hair.
> Some Africans are Negroes.
> ∴ Some Africans have woolly hair.
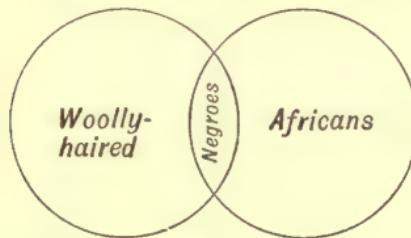
When a premiss is negative, it gives us no clue at all to the relative extent of the terms employed.

§ 267. Outside books on logic an argument is seldom found stated in full syllogistic form. The conclusion is generally stated first and then one of the premisses thrown in as a reason for it.

Problema or Quaestio.

§ 268. When the conclusion is put as a question open to discussion, it is called a *Problema* or *Quaestio*. The solution or answer is supplied by the premisses, thus—

> Are the Chinese capable of progress or are they not?
> They are: for they are rational beings, and all rational
>   beings are capable of progress.

Enthymeme.

§ 269. An Enthymeme is properly a rhetorical syllogism, which may or may not be a valid argument. The name however is very commonly used now for an argument defectively expressed.

In an Enthymeme of the First Order the major premiss is suppressed.

In an Enthymeme of the Second Order the minor premiss is suppressed.

In an Enthymeme of the Third Order the conclusion is suppressed.

The Problematic Enthymeme.

§ 270. An enthymeme of the first or second order may be problematically expressed. Thus to the question 'Does A distribute its subject or not?' we may reply 'Yes, because it is universal' or 'Yes, because all universal propositions do.' By the former answer we are giving an enthymeme of the first, by the latter one of the second order. Either answer is incomplete without the other. The logical difference between the two is that in the one we are giving the major premiss, in the other the minor premiss of the syllogism; in the one the principle, in the other its application.

CHAPTER XX.

Of Mood and Figure.

§ 271. Syllogisms may differ from one another in two ways—

(1) in Mood,

(2) in Figure.

§ 272. Mood is a difference among syllogisms depending on the quantity and quality of the propositions of which they are made up.

§ 273. Figure is a difference among syllogisms depending upon the position of the terms in the propositions.

§ 274. A syllogism consisting of the same kind of propositions, say EIO, may have its terms arranged differently : so that the same mood may be in different figures.

There are 64 possible moods.

There are 4 possible figures.

∴ There are 256 possible forms of syllogism.

§ 275. That there are 64 possible moods may be seen as follows.

Suppose the premisses to be AA. These may yield four different conclusions, giving us the four moods AAA, AAE, AAI, AAO. But instead of having A for the

minor premiss, we might have E or I or O, bringing up the number of moods to 4 × 4. These all have A for the major premiss. There will be the same number with E, I, and O. Altogether then the number of moods is 4 × 4 × 4 × 4 = 256.

§ 276. That there are four possible figures may be seen as follows.

The position of the terms in the conclusion is fixed by definition. We have therefore only to consider their position in the premisses. But as there are two terms in a premiss, of which the middle is always one, the position of the middle term fixes that of the other two. We have therefore only to consider the position of the middle term. Now the middle term must either occupy the same position in both premisses or not. If it does occupy the same position in both, it must either be subject in both or predicate in both; if it does not occupy the same position in both, it must either be subject in the major and predicate in the minor or else predicate in the major and subject in the minor.

§ 277. The First Figure is that arrangement of terms in a syllogism in which the middle term is subject in the major premiss and predicate in the minor.

The Second Figure is that arrangement of terms in a syllogism in which the middle term is predicate in both premisses.

The Third Figure is that arrangement of terms in a syllogism in which the middle term is subject in both premisses.

The Fourth Figure is that arrangement of terms in a syllogism in which the middle term is predicate in the major premiss and subject in the minor.

Let P = major term (predicate of the conclusion).

   S = minor term (subject of the conclusion).

   M = middle term.

Figure I.	Figure II.	Figure III.	Figure IV.
M—P.	P—M.	M—P.	P—M.
S—M.	S—M.	M—S.	M—S.
∴ S—P.	∴ S—P.	∴ S—P.	∴ S—P.

Figure I.

E.   No *horned animals* are carnivorous.

I.   Some quadrupeds *have horns.*

O. ∴ Some quadrupeds are not carnivorous.

Figure II.

E.   No carnivorous animals *have horns.*

I.   Some quadrupeds *have horns.*

O. ∴ Some quadrupeds are not carnivorous.

Figure III.

E.   No *horned animals* are carnivorous.

I.   Some *horned animals* are quadrupeds.

O. ∴ Some quadrupeds are not carnivorous.

Figure IV.

E.   No carnivorous animals *have horns.*

I.   Some *horned animals* are quadrupeds.

O. ∴ Some quadrupeds are not carnivorous.

CHAPTER XXI.

Of the Canon of Reasoning.

§ 278. WHATEVER can be affirmed or denied of a whole class can be affirmed or denied of everything contained in that class.

The above axiom applies directly only to the first figure. Hence

the first figure is called the Perfect Figure,
the rest are called the Imperfect Figures.

The way in which the axiom is stated is convenient, as it makes the principle intuitively manifest, but it has the drawback of regarding the predication in the minor premiss as being made in extension, i.e. as consisting in inclusion in a class.

§ 279. If we wish to express the principle intensively, we may do so as follows—

Whatever has certain attributes has also the attributes which invariably accompany them.

Thus, if a whale has the attribute of suckling its young, it has also the attribute of warm-bloodedness, which is found invariably to accompany the other.

Or, beginning from the other end, we may put the canon thus—

> An attribute of an attribute of anything is an attribute of the thing itself.

> Nota notae est nota rei ipsius.

Either mode of statement is under the disadvantage of taking the subject of the major premiss in intension.

§ 280. As a subject is always used in extension and a predicate generally in intension, the proper statement of the principle is unavoidably clumsy—

> If a thing has a certain attribute and all the things which possess that attribute have another attribute also, then the thing in question possesses that other attribute.

> All whales suckle their young.
> All animals that suckle their young have warm blood.

∴ All whales have warm blood.

§ 281. By Aristotle himself (*Cat.* 3, § 1) the principle was expressed in a neutral form thus—

> 'Whatever is stated of the predicate will be stated also of the subject.'

But there are two assumptions here, namely that the statement about the predicate is made universally and that the original proposition is affirmative. Let us therefore enlarge his statement as follows—

> Whatever is affirmed or denied universally of the predicate of an affirmative proposition may be affirmed or denied also of the subject.

The student should bear in mind that, as the syllogism is usually stated, the affirmative proposition spoken of is the second premiss.

In the above example it is the first.

The Three Axioms of Mediate Inference.

§ 282. Instead of a single canon applying only to the perfect figure modern logicians tend to substitute three axioms which apply to all figures alike.

(1) If two terms agree with the same third term, they agree with one another.

(2) If one term agrees and another disagrees with the same third term, they disagree with one another.

(3) If two terms disagree with the same third term, they may or may not agree with one another.

§ 283. The first of these axioms is the principle of all affirmative conclusions;

the second is the principle of all negative conclusions;

the third points out the conditions under which no conclusion can be drawn.

§ 284. Even more than the Dictum *de Omni et Nullo* these axioms labour under the defect of representing predication as a mere matter of extension. For by 'agreement of terms' can only be meant coincidence in extension.

When it is said that two terms agree with the same third term, it must be understood that they agree with the same part of it.

Here are two terms which appear to agree with the same third term—

> All badgers are bald,
> All babies are bald,

and yet no one would contend that they agree with one another.

Here again are two terms of which one appears to agree and the other to disagree with the same third term—

> All birds lay eggs,
> No lizards are birds,

and yet we cannot draw the conclusion, 'No lizards lay eggs,' nor even 'Some lizards do not lay eggs.'

The fact is that these axioms do not of themselves enable us to distinguish a good conclusion from a bad one. They only do so, if we assume along with them the rules for the distribution of terms together with the rule of syllogism, that no term must be distributed in the conclusion which was not distributed in the premises. Out of the premises last given we may extract the conclusion, 'Some things which lay eggs are not lizards,' if we duly keep before our minds the laws of distribution.

CHAPTER XXII.

Of the General Rules of Syllogism.

§ 285. I. PRIMARY.

(1) A syllogism must consist of three propositions only.

(2) A syllogism must consist of three terms only.

(3) The middle term must be distributed in one or both of the premisses.

(4) No term must be distributed in the conclusion which was not distributed in the premisses.

(5) If both premisses be affirmative, the conclusion must be affirmative, and vice versa.

II. DERIVATIVE.

(6) If one premiss be negative, the conclusion must be negative, and vice versa.

(7) Two negative premisses prove nothing.

(8) Two particular premisses prove nothing.

(9) If one premiss be particular, the conclusion must be particular.

§ 286. The first two of the above rules deal with the structure of the syllogism and partake of the nature of definition.

The second two deal with the distribution of terms.

The next three relate to the quality of the propositions.

The last two relate to the quantity of the propositions.

The first five rules are of the nature of definitions and axioms and do not admit of proof; the remaining four can be proved from them.

§ 287. (1) A syllogism may be defined as the comparison of two propositions by means of a third.

∴ It must consist of three propositions only.

If there be more than two premisses, there is more than one syllogism; if there be less, there is no syllogism, but either an immediate inference or a mere asseveration.

The latter case comes under the fallacy of 'begging the question,' which means giving a statement as a reason for itself.

Smoking is unwholesome.

Why?	All S is P.
Because it is.	∴ All S is P.

(2) A syllogism may be defined as the comparison of two terms by means of a third.

∴ It must consist of three terms only.

If there be more than three terms, there is either no syllogism or more than one; if there be less, there is no syllogism. The following is vituperation, not argument.

You are a fool.	S—P.
You are you.	S—S.
∴ You are a fool.	∴ S—P.

This also comes under the fallacy of 'begging the question.' In conversation it might appear as follows—

> You are a fool.
> Why?
> Because you are yourself.

The violation of this rule in the way of excess is called the fallacy of Four Terms. It ought to be called the fallacy of More than Three Terms, since there may be as many as six

His views are true.	All M is P.
My views are like his.	All S is N.
∴ My views are true.	∴ All S is P.

This fallacy also arises whenever an ambiguous word is used in its different senses for one of the three terms.

The next two rules guard against the two fallacies which are fatal to most syllogisms whose constitution is unsound.

(3) If the mean be not compared in its whole extent with either of the extremes, we may be referring to one part of it in one premiss and to quite another part of it in the other, so that there will be no real mean at all.

All rash men are confident.	All P is M̆.
All brave men are confident.	All S is M̆.
∴ All brave men are rash.	∴ All S is P.

The violation of this rule is known as the fallacy of Undistributed Middle.

(4) If a term be undistributed in the premiss and distributed in the conclusion, we are arguing from the

part to the whole, which is never a safe form of reasoning.

The violation of this rule is known as the fallacy of Illicit Process.

ILLICIT PROCESS OF THE MAJOR TERM.

All Oxford men are educated.	All M is $\breve{P}$.
No artisans are Oxford men.	No S is M.
∴ No artisans are educated.	∴ No S is $\bar{P}$.

ILLICIT PROCESS OF THE MINOR TERM.

No liar can be trusted.	No M is P.
Some liars are entertaining.	Some M is $\breve{S}$.
∴ No entertaining person can be trusted.	∴ No $\bar{S}$ is P.

The next three rules are the three axioms of deductive inference confined to the sphere within which they are really operative.

(5) Two judgements of agreement may give ground for a new judgement of agreement, but not for one of disagreement; and, conversely, a judgement of agreement may arise out of two judgements of agreement, but never out of a judgement of agreement combined with one of disagreement.

There is not much semblance of argument about the following—

All prime ministers are proud.	All M is $\breve{P}$.
All prime ministers are paid.	All M is $\breve{S}$.
∴ No paid persons are proud.	∴ No $\bar{S}$ is $\bar{P}$.

Nor will this stand examination—

> Some shell-less animals are snails; for all slugs are
> shell-less and no snails are slugs.

> > No P is M.
> > All M is S.
> > ∴ Some S is P.

Both of the above examples violate Rule 5; the former involves a double illicit process as well.

§ 288. (6) A judgement of agreement combined with one of disagreement may give ground for a new judgement of disagreement, but not for one of agreement; and, conversely, a judgement of disagreement may arise out of a judgement of agreement combined with one of disagreement, but never out of two judgements of agreement.

This rule in its two parts is the contrapositive of 5, the second part of 6 being the contrapositive of the first part of 5, and the first part of 6 the contrapositive of the second part of 5.

> If both premisses be affirmative, the conclusion must be affirmative.

> ∴ If the conclusion be not affirmative, both premisses are not affirmative.

> i.e. If the conclusion be negative, one premiss must be negative.

> If the conclusion be affirmative, both premisses must be affirmative.

∴. If both premisses be not affirmative, the conclusion
     is not affirmative.

   i.e. If one premiss be negative, the conclusion must
     be negative.

Rule 5 may be put in the form of a U proposition—
All the cases of wholly affirmative premisses are all the
cases of affirmative conclusion.

This admits of two inferences being drawn from it.

   (*a*) All cases of partly negative premisses are cases
     of negative conclusion.

   (*b*) All cases of negative conclusion are cases of
     partly negative premisses.

     All S is all P.

  ∴. All not-S is not-P (Contrapositive without
                 conversion).

  and All not-P is not-S (Converse by Contra-
                 position).

Violations of Rule 6 are necessarily also violations
of Rule 5.

Here is an apparent instance.

     What does not twinkle is near.

     No planets twinkle.

  ∴. All planets are near.

But it is not in syllogistic form, since there are four
terms. It is really an instance of reasoning with a
negative middle term.

     Non-twinkling stars are near.

     All planets are non-twinkling stars.

  ∴. All planets are near.

§ 289. (7) Two judgements of disagreement give no ground for any new judgement at all. If all that we know of Shadrach and Abednego is that they are not like Meshech, we cannot tell whether they are like each other or not.

No fishes are mammals.	No P is M.
No newts are mammals.	No S is M.
∴ No newts are fishes.	∴ No S is P.

Proof of the rule that two negative premisses prove nothing.

If both premisses be negative, they are either both universal or not.

Let them be both universal, namely EE.

Then, in whatever figure they be put, they may by the aid of obversion and, where necessary, of conversion be brought into the form

All P is not-M,
All S is not-M

—which involves undistributed middle.

Next let them not be both universal.

Then, since two negative premisses even when both universal can give no conclusion, *a fortiori* they can give none when one or both are particular [1].

It must be borne in mind that this like the rest is a rule

[1] This proof is taken from De Morgan and from Keynes, *Formal Logic*, § 117, 1st edit.

of syllogism. It ought not to be taken to mean that we can never extract a conclusion from two propositions in negative form. This may easily be done. The following, for instance, is very good reasoning.

> None who do not believe rightly can be saved.
> No men at all believe rightly.
> ∴ No men at all can be saved.

> > No not-M is P.
> > No S is M.
> > ∴ No S is P.

Only it is not in syllogistic form, as it has four terms. If we wish to make it a syllogism, we must read the minor as an affirmative—

> > All S is not-M.

Again this is good reasoning—

No monkeys are men.	No M is P.
No monkeys are without hands.	No M is S.
∴ Some creatures with hands are not	
men.	∴ Some not-S is not-P.

But, as before, it has four terms.
As a syllogism it appears in this form—

> > All M is not-P.
> > All M is not-S.
> > ∴ Some not-S is not-P.

§ 290. (8) This and the following rule may be proved from those which relate to the distribution of terms.

Proof of the rule that two particular premisses prove nothing.

If both premisses be particular, they must be either both affirmative or both negative or one affirmative and one negative.

There are then three cases—II, OO, and IO or OI. But II premisses distribute no term at all.

∴ The middle term must be undistributed.

∴ There is no conclusion.

OO are excluded by Rule 7.

IO or OI distribute only one term, which must be the middle term (Rule 3).

∴ The major term is undistributed in the premisses.

But one of the premisses being negative, the conclusion must be negative.

And every negative proposition distributes its predicate.

∴ The major term must be distributed in the conclusion.

∴ There would be an illicit process of the major.

∴ There is no conclusion.

§ 291. (9) *Proof of the rule that, if one premiss be particular, the conclusion must be particular.*

If one premiss be particular, the other must be universal (by Rule 8).

There are then four combinations possible—

A with I, A with O, E with I, E with O.

In AI or IA premisses only one term is distributed.

This must be the middle term (Rule 3).

∴ The minor term is undistributed in the premisses.

∴ It must be undistributed in the conclusion (Rule 4).

∴ The conclusion must be particular.

In the combination of A with O and likewise in that of E with I there are two terms distributed.

One of these must be the middle term (Rule 3).

∴ There is only one of the extremes distributed in the premisses.

But one premiss is negative.

∴ The conclusion must be negative (Rule 6).

∴ The major term must be distributed in the conclusion.

∴ The one extreme that is distributed in the premisses must be the major term.

∴ The minor term is undistributed in the premisses.

∴ It cannot be distributed in the conclusion (Rule 4).

∴ The conclusion is particular.

The combination of E with O is excluded by Rule 7.

§ 292. Subtracting the first two rules as being definitions and the last four as being derivative, we are left with an irreducible residuum in 3, 4, and 5. Rule 5 has always hitherto been taken for granted instead of being expressed.

§ 293. The following mnemonics contain all our rules except 1 and 5.

> Distribuas medium, nec quartus terminus adsit ;
> Utraque nec praemissa negans, nec particularis ;
> Sectetur partem conclusio deteriorem,
> Et non distribuat, nisi cum praemissa, negetve.

The rule that 'the conclusion must follow the weaker part' covers 6 and 9.

§ 294. All the rules of syllogism are implicitly at least contained in the statement of the Canon of Reasoning [1]—

> 'Whatever is affirmed or denied universally of the predicate of an affirmative proposition may be affirmed or denied also of the subject.'

This statement implies—

(1) 3 propositions, namely, the affirmative proposition (minor premiss), the universal predication (major premiss), the predication about the subject (conclusion);

(2) 3 terms, namely, the subject of the affirmative proposition (minor term), the predicate of the same (middle term), the term which is predicated universally of its subject (major term);

(3) the distribution of the middle term, since the predication is made of it universally;

(4) the absence of illicit process; for, if the minor were used illicitly, there would be a predication about something more than the subject, and, if the major were so used, it could only be by denying of the subject what had been affirmed of the predicate, which is against the Canon;

(5) that one premiss is affirmative, since it is so given;

(6) that, if one premiss be negative, the conclusion must be negative, since what is denied of the predicate is to be denied also of the subject;

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[1] Here I am under obligations to Mr. Keynes, *Formal Logic*, § 126, 1st edit.

(7) that both premisses cannot be negative, since one is given as affirmative ;

(8) that both premisses cannot be particular, since one is given as universal ;

(9) that, if one premiss be particular, the conclusion must be particular, since the one premiss must be the minor, and if that is particular while the conclusion is universal, there will be a predication about something more than the subject.

§ 295. The rules relating to the quality of the form (5, 6, 7) should be compared and contrasted with the following rules relating to the quality of the matter.

(1) If both premisses be true, the conclusion must be true, but not vice versâ.

(2) If the conclusion be false, one or both premisses must be false, but not vice versâ.

Instances of true conclusion from false premisses.

All men are bipeds.	All men are quadrupeds.
All birds are men.	All cows are men.
∴ All birds are bipeds.	∴ All cows are quadrupeds.

Since truth can never give rise to falsehood and falsehood may give rise to truth, we may hope for a triumph of truth in the long run.

CHAPTER XXIII.

Of the Determination of the Legitimate Moods.

§ 296. WE found that there were 64 possible moods.

By the application of the preceding rules these are reduced to the following—

11 LEGITIMATE MOODS.

AAA. AAI. AEE. AEO. AII. AOO.
EAE. EAO. EIO.
IAI.
OAO.

The reader may satisfy himself of this result by writing down the 64 possible moods and then striking out those which violate any of the nine rules of syllogism. This is a good exercise.

§ 297. But a much neater way of attaining the same end is first to find what pairs of premises are legitimate in accordance with Rules 7 and 8, and then to see what conclusions can be drawn from them.

If the major be A, there is nothing to restrict us in our choice of a minor.

If the major be E, the minor must be affirmative (Rule 7).

If the major be I, the minor must be universal (Rule 8).

If the major be O, the minor must be universal (Rule 8) and affirmative (Rule 7).

Hence there result nine legitimate pairs of premisses.

<div align="center">

AA. AE. AI. AO.

EA. EI.

IA. (IE).

OA.

</div>

But though the union of an I major with an E minor is legitimate enough, it is nevertheless doomed to sterility.

§ 298. *Proof that* IE *premisses can give no conclusion.*

If possible let there be a conclusion.

Then it must be negative (Rule 6).

And every negative proposition distributes its predicate.

∴ The major term must be distributed in the conclusion.

But the major premiss I does not distribute either term.

∴ There must be an illicit process of the major.

§ 299. Disregarding IE therefore and remembering that wherever we can draw a universal conclusion we can also draw a particular, we arrive at the eleven moods already given.

§ 300. A Subaltern Mood is one which draws a particular conclusion from premisses which warrant a universal. It is so called because its conclusion might be inferred by

subalternation from that of the syllogism with a universal conclusion.

The moods AAI, AEO, EAO are subaltern to AAA, AEE, EAE respectively.

A subaltern mood is also called a syllogism with a weakened conclusion.

CHAPTER XXIV.

Of the Special Rules of the Four Figures.

§ 301. Our next task is to determine how many of the eleven legitimate moods are valid in the four figures.

With a view to doing so we lay down the following

Special Rules of the Four Figures.

Figure I.

Rule 1.   The minor premiss must be affirmative.

Rule 2.   The major premiss must be universal.

Figure II.

Rule 1.   One premiss must be negative.

Rule 2.   The conclusion must be negative.

Rule 3.   The major premiss must be universal.

Figure III.

Rule 1.   The minor premiss must be affirmative.

Rule 2.   The conclusion must be particular.

Figure IV.

Rule 1.   If the major premiss be affirmative, the minor must be universal.

Rule 2.   If the minor premiss be affirmative, the conclusion must be particular.

Rule 3.   If either premiss be negative, the major must be universal.

§ 302. These special rules follow from the general rules of syllogism taken in connexion with the position of the terms in the different figures.

The special rules of the first figure are really a repetition of the Canon of Reasoning itself; nevertheless they can be proved like the rest from the general rules.

Proof of the Special Rules of the Four Figures.

Figure I.

Proof of Rule 1. *The minor premiss must be affirmative.*

If possible, let the minor premiss be negative. M — P
Then the major must be affirmative (Rule 7) S — M
and the conclusion must be negative (Rule 6). ∴ S — P
But no affirmative proposition distributes its predicate and every negative proposition does.

And the major term is predicate both in the major premiss and in the conclusion.

∴ The major term will be distributed in the premiss
and undistributed in the conclusion,

i.e. there will be an illicit process of the major term.

And this follows from the supposition that the minor premiss is negative.

∴ The minor premiss must be affirmative.

Proof of Rule 2. *The major premiss must be universal.*

Since the minor premiss is affirmative, the middle term is not distributed there.

∴ It must be distributed in the major premiss (Rule 3).

But it is subject in the major premiss.

∴ The major premiss must be universal.

Figure II.

Proof of Rule **1**. *One premiss must be negative.*

If not, the middle term would be undis-   P — M
      tributed.   S — M

∴ One premiss must be negative.   ∴ S — P

Proof of Rule **2**. *The conclusion must be negative.*

One of the premisses is negative.

∴ The conclusion must be negative (Rule 6).

Proof of Rule **3**. *The major premiss must be universal.*

The conclusion is negative.

∴ The major term must be distributed in the con-
    clusion.

∴ It must be distributed in the major premiss (Rule 4).

But it is subject in the major premiss.

∴ The major premiss must be universal.

Figure III.

Proof of Rule **1**. *The minor premiss must be affirmative.*

The proof is the same as in the first figure,   M — P
depending on the position of the major term,   M — S
which is the same in both.   ∴ S — P

Proof of Rule **2**. *The conclusion must be particular.*

The minor premiss is affirmative.

∴ The minor term is undistributed in the premiss.

∴ It is undistributed in the conclusion (Rule 4).

∴ The conclusion must be particular.

Figure IV.

Proof of Rule 1. *If the major premiss be affirmative, the minor must be universal.*

If the major premiss were affirmative and the minor particular, the middle term would be undistributed in both.

<div style="text-align:right">

P — M
M — S
∴ S — P

</div>

∴ If the major premiss be affirmative, the minor must
  be universal.

Proof of Rule 2. *If the minor premiss be affirmative, the conclusion must be particular.*

If the minor premiss be affirmative, the minor term is undistributed there.

∴ It must be undistributed in the conclusion.

∴ If the minor premiss be affirmative, the conclusion
  must be particular.

Proof of Rule 3. *If either premiss be negative, the major must be universal.*

If either premiss be negative, the conclusion will be negative (Rule 6).

∴ The major term will be distributed in the con-
  clusion.

∴ It must be distributed in the major premiss (Rule 4).

∴ If either premiss be negative, the conclusion must
  be universal.

CHAPTER XXV.

Of the Determination of the Valid Moods in the Four Figures.

§ 303. WE found that there were eight pairs of premisses which could yield legitimate conclusions—

AA. AE. AI. AO.
EA. EI.
IA.
OA.

which, when the conclusions were appended to them, gave us the following—

11 LEGITIMATE MOODS.

AAA. AAI. AEE. AEO. AII. AOO.
EAE. EAO. EIO.
IAI.
OAO.

§ 304. How many of these moods are valid in the different figures? This question may be answered in two ways—

    (1) by taking the eight pairs of premisses and appending conclusions to them in accordance with the special rules for the four figures;

(2) by applying the special rules for the four figures
to the fully formed moods themselves.

We will adopt the latter process, leaving it to the
student to verify the results by the former one.

Moods that are valid in Figure I.

§ 305. The rule that the minor premiss must be
affirmative invalidates AEE, AEO, AOO.

The rule that the major premiss must be universal
invalidates IAI, OAO.

Thus we are left with six moods that are valid in the
first figure, namely—

AAA. EAE. AII. EIO. AAI. EAO.

Moods that are valid in Figure II.

§ 306. The rule that the conclusion must be negative
(which implies the preceding rule that one or other
premiss must be negative) invalidates AAA, AAI, AII,
IAI.

The rule that the major premiss must be universal
invalidates OAO.

Thus we are left with six moods that are valid in the
second figure, namely—

EAE. AEE. EIO. AOO. EAO. AEO.

Moods that are valid in Figure III.

§ 307. The rule that the minor premiss must be
affirmative invalidates AEE, AEO, AOO (as in Figure I).

The rule that the conclusion must be universal
invalidates AAA, EAE.

Thus we are left with six moods that are valid in the third figure, namely—

> AAI.　IAI.　AII.　EAO.　OAO.　EIO.

> *Moods that are valid in Figure IV.*

§ 308. The rule that, if the major premiss be affirmative, the minor must be universal invalidates AII, AOO.

The rule that, if the minor premiss be affirmative, the conclusion must be particular invalidates AAA, EAE.

The rule that, if either premiss be negative, the major must be universal invalidates OAO.

Thus we are left with six moods that are valid in the fourth figure, namely—

> AAI.　AEE.　IAI.　EAO.　EIO.　AEO.

§ 309. Putting the above results together we obtain the following—

Twenty-four Valid Combinations of Mood and Figure.

FIGURE I.　AAA.　EAE.　AII.　EIO.　[AAI.　EAO.]
FIGURE II.　EAE.　AEE.　EIO.　AOO.　[EAO.　AEO.]
FIGURE III.　AAI.　IAI.　AII.　EAO.　OAO.　EIO.
FIGURE IV.　AAI.　AEE.　IAI.　EAO.　EIO.　[AEO.]

§ 310. The five of these that are inclosed in brackets are subaltern moods or syllogisms with weakened conclusions (§ 300). Such a syllogism is valid enough, but it is practically worthless. Take for instance AAI in Figure I. From the premisses—

> All men are animals,
> 　and All animals are mortal,

we can draw the conclusion—

> All men are mortal.

Who then would content himself with the lesser inference—

Some men are mortal,

which is already contained in the other?

§ 311. Subtracting then the five subaltern moods we are left with nineteen combinations of mood and figure (often loosely called 'moods') which are at once valid and useful. These are indicated by the vowels in the following mnemonic lines—

Barbara, Celarent, Darii, Ferioque prioris;
Cesare, Camestres, Festino, Baroco secundae ;
tertia **Darapti, Disamis, Datisi, Felapton,**
Bocardo, Ferison habet; quarta insuper addit
Bramantip, Camenes, Dimaris, Fesapo, Fresison.
Quinque subalterni, totidem generalibus orti,
nomen habent nullum, nec, si bene colligis, usum.

The last two lines may be translated thus—' The five subaltern moods, which are derived from the same number of moods with universal conclusions, have no name and, if you draw the conclusion rightly, no use.' The student however should never lose sight of the fact that there are these subaltern moods, as it affects the answer to a good many questions. He may supply names for himself by merely changing the last vowel, e.g. the subaltern to *Camenes* may be called *Camenos* [1].

---

[1] Such a name will indicate the mood and figure, but will not serve as a symbol for reduction.

CHAPTER XXVI.

Of the Specialities of the Four Figures.

§ 312. The special rules of the four figures, which have already been given, may all be educed from an inspection of the mnemonic lines, e.g. by running through the line for the second figure—'**Cesare, Camestres, Festino, Baroco** secundae,' we can satisfy ourselves that the conclusion is always negative.

§ 313. Another way of arriving at the same result is to write down the six moods that are valid in each of the four figures, arranging them in vertical columns, so that all the major premisses, the minor premisses, and the conclusions may be read together in horizontal lines. Then, if anything can be predicated universally of these, that will be a special rule of the given figure.

Figure I.

A	E	A	E	A	E	Universal.
A	A	I	I	A	A	Affirmative.
A	E	I	O	I	O	

There is no rule about the conclusion in Figure I since there is nothing which can be predicated of it universally.

Figure II.

E	A	E	A	E	A	Universal.
A	E	I	O	A	E	One or other premiss negative.
E	E	O	O	O	O	Negative.
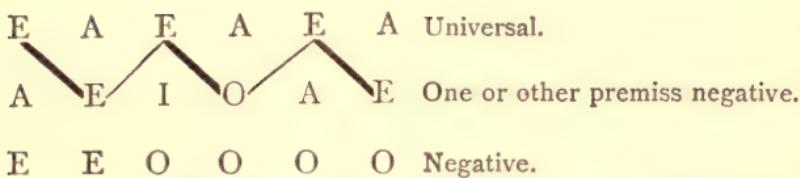
The rule that one or other premiss must be negative has to be educed from a comparison of the two premisses.

Figure III.

A	I	A	E	O	E	
A	A	I	A	A	I	Affirmative.
I	I	I	O	O	O	Particular.

There is no rule about the major premiss in Figure III, since there is nothing which can be predicated of it universally.

Figure IV.

A	A	I	E	E	A	
A	E	A	A	I	E	
I	E	I	O	O	O	

If the major be affirmative, the minor must be universal.

If the minor be affirmative, the conclusion must be particular.

If the conclusion be negative, the major must be universal.
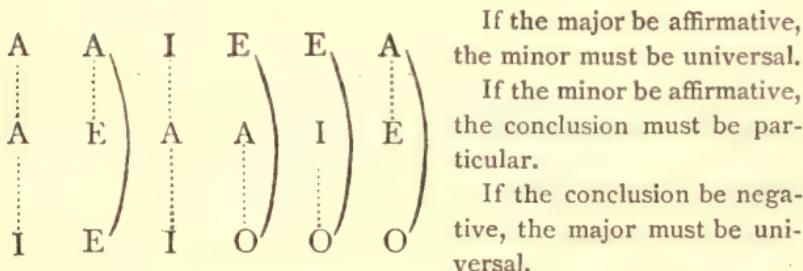
The only universal statements that can be made about the fourth figure are that neither of the premisses can be a particular negative, and that the conclusion cannot be a universal affirmative. These would serve to invalidate

only three out of the eleven legitimate moods. Hence recourse must be had in this figure to a comparison between the propositions. This is why the hypothetical form of statement is adopted.

The reader will observe that the form of the third rule of Figure IV has been altered for convenience.

§ 314. The following points should be specially noted.

The first figure proves any kind of conclusion, and is the only one which can prove A.

The second figure proves only negatives.

The third figure proves only particulars.

The fourth figure proves any conclusion except A.

Reasons why the First is considered the Perfect Figure.

§ 315. (1) It alone complies directly with the Canon of Reasoning.

(2) It suffices to prove every kind of conclusion, and is the only figure in which a universal affirmative conclusion can be established.

(3) It is only in a mood of this figure that the major, middle, and minor terms stand in their relative order of extension.

(4) It is only in this figure that both subject and predicate in the conclusion are subject and predicate also in the premisses.

§ 316. The fact that the first figure alone proves A is sufficient of itself to assign it the primacy. As scientific truths assume this form, the first figure is also called the Scientific Figure.

Proof that A can only be established in the first figure.

Let the conclusion be a universal affirmative—

<div style="text-align:center">All S is P.</div>

Then by Rule 5 (§ 285) both premisses must be affirmative.

Now, since the conclusion is universal, the minor term must be distributed there.

∴ It must be distributed in the minor premiss.

∴ It must be subject in the minor premiss, since no affirmative distributes its predicate.

∴ The middle term must be predicate in the minor premiss.

But the minor premiss is affirmative.

∴ The middle term is undistributed in the minor premiss.

∴ The middle term must be distributed in the major premiss.

But the major premiss is affirmative.

∴ The middle term must be subject in the major premiss.

But when the middle term is subject in the major premiss and predicate in the minor, we have what is known as the first figure.

∴ A can only be established in the first figure.

Special Canons of the Imperfect Figures.

§ 317. The intrinsic superiority of the first figure as a mould in which to exhibit the cogency of reasoning is in

no way more manifest than from the difficulty of providing the imperfect figures with canons of their own.

The fourth figure baffles all such attempts, but the following have been suggested for the second and the third.

Canon of the Second Figure or Dictum de Diverso.

If one term is contained in, and another excluded from, a third term, they are mutually excluded.

Canon of the Third Figure or Dictum de Exemplo et de Excepto.

Two terms which contain a common part partly agree, or, if one contains a part which the other does not, they partly differ.

§ 318. The neat and compendious form, in which the *Dictum de Diverso* has been expressed, is unfortunately deceptive. For by 'term' is meant 'term or part of a term,' and this fact makes havoc of the whole statement. The canon, fully expressed, would be intolerably prolix—

If one term is wholly or partly contained in, and another wholly excluded from, the same third term, the one is wholly (**Cesare**) or partly (**Festino**) excluded from the other; and if one term is wholly contained in, and another wholly or partly excluded from, the same third term, the one is wholly (**Camestres**) or partly (**Baroco**) excluded from the other.

The *Dictum de Exemplo* is unexceptionable. The part common to P and S may be 'all M,' as in **Darapti**, or 'some M,' as in **Disamis** and **Datisi**.

Darapti.	Disamis.	Datisi.
All M is P.	Some M is P.	All M is P.
All M is S.	All M is S.	Some M is S.
∴ Some S is P.	∴ Some S is P.	∴ Some S is P.

The statement of the canon does not exclude the conclusion in all three cases, Some P is S: but then neither do the rules of syllogism.

The *Dictum de Excepto* however is not so satisfactory. It leaves it open to us to draw the conclusion Some P is not S, which would in all cases involve an illicit process, as well as the conclusion Some S is not P, which is sound.

Felapton.	Bocardo.	Ferison.
No M is P.	Some M is not P.	No M is P.
All M is S.	All M is S.	Some M is S.
∴ Some S is not P.	∴ Some S is not P.	∴ Some S is not P.

The whole canon, it would seem, might be satisfactorily stated as follows—

Dictum de Exemplo et de Excepto.

Two terms which contain a common part partly agree, or, if the second contains a part ('all M' in **Felapton** and **Bocardo**, 'some M' in **Ferison**) which the first does not, the first may be partly denied of the second.

§ 319. The foregoing statement of the canons of the second and third figures is made from the point of view of extension, like the usual one of the *Dictum de Omni et Nullo.*

If we adopt the point of view which regards the subject as used in extension and the predicate commonly in intension, we shall gain in truth, and, so far as concerns the second figure, not lose in clearness.

Canon of the Second Figure or Dictum de Diverso.

If a subject has an attribute which a class has not, or vice versâ, the subject does not belong to the class.

Canon of the Third Figure or Dictum de Exemplo et de Excepto.

If a certain attribute can be affirmed or denied of the whole or part of a class, it may also be affirmed or denied of some of the things which exhibit another attribute belonging to the whole or part of that class.

To enable the reader the more easily to verify the last somewhat complicated statement, we here append concrete examples in the same matter of the six moods of the third figure. It should be added that, here as elsewhere, a singular term may take the place of a class-name used in its full extent.

Darapti.

All selfish men are shortsighted.
All selfish men are bad.

∴ Some bad men are shortsighted.

Disamis.

Some selfish men are shortsighted.
All selfish men are bad.

∴ Some bad men are shortsighted.

Datisi.

All selfish men are shortsighted.
Some selfish men are bad.

∴ Some bad men are shortsighted.

Felapton.

No selfish men are longsighted.
All selfish men are bad.

∴ Some bad men are not longsighted.

Bocardo.

Some selfish men are not longsighted.
All selfish men are bad.

∴ Some bad men are not longsighted.

Ferison.

No selfish men are longsighted.
Some selfish men are bad.

∴ Some bad men are not longsighted.

SPECIAL USES OF THE FOUR FIGURES.

§ 320. Lambert's statement on this subject has gained general acceptance. Roughly it is as follows.

The first figure is useful for proving the properties of a thing.

The second figure is useful for proving distinctions between things.

The third figure is useful for proving instances or exceptions.

The fourth figure is useful for proving the species of a genus.

FIGURE I.

M is or is not P.

S is M.

∴ S is or is not P.

We prove that S has or has not the property P by predicating of it M, which we know to possess or not to possess that property.

All trees bear fruit.

The pine is a tree.

∴ The pine bears fruit.

No Aryans are black men.

All Parsees are Aryans.

∴ No Parsees are black men.

FIGURE II.

P is M.	P is not M.
S is not M.	S is M.
∴ S is not P.	∴ S is not P.

We establish the distinction between S and P by showing that P has an attribute which S is devoid of, or is devoid of an attribute which S has.

All fishes are coldblooded.	No fishes give milk.
A whale is not coldblooded.	A whale gives milk.
∴ A whale is not a fish.	∴ A whale is not a fish.

FIGURE III.

M is P.	M is not P.
M is S.	M is S.
∴ Some S is P.	∴ Some S is not P.

We produce instances of S being P by showing that S and P meet, at all events partially, in M. Thus, if we wish to produce an instance of the compatibility of great learning with original powers of thought, we might say—

Sir William Hamilton was an original thinker.

Sir William Hamilton was a man of great learning.

∴ Some men of great learning are original thinkers.

Or we might urge an exception to the supposed rule about Scotchmen being deficient in humour under the same figure, thus—

Sir Walter Scott was not deficient in humour.

Sir Walter Scott was a Scotchman.

∴ Some Scotchmen are not deficient in humour.

FIGURE IV.

P is M.	All P is M.	No P is M.
All M is S.	No M is S.	M is S.
∴ Some S is P.	∴ No S is P.	∴ Some S is not P.

When the conclusion is affirmative, we prove that P is a species of S by showing that P falls under M, which itself falls under S.

Bramantip and Dimaris.

Boers are white natives of Africa.

All white natives of Africa are Africanders.

∴ Some Africanders are Boers.

When the conclusion is a universal negative, we prove that P is no species of S by showing that all P falls under M, which we know to be excluded from S.

Camenes.

All tomatos are fruits.
No fruits are roots.
∴ No roots are tomatos.

When the conclusion is a particular negative, we prove that, whether P be a species of S or not, there are at all events other species of S besides P. This we do by showing that P is excluded from M, which we know to fall, wholly or partially, under S.

Fesapo and Fresison.

No fishes are snakes.
Snakes can swim.
∴ Some swimming things are not fishes.
No Englishborn are white natives of Africa.
White natives of Africa are Africanders.
∴ Some Africanders are not Englishborn.

§ 321. The multiplicity of method in the fourth figure throws light upon the difficulty of formulating a single canon for it. Not to leave it however without some evidence of its own, we will lay down the following

Three Axioms of the Fourth Figure.

(1) Whatever is affirmed of a whole term may have partially affirmed of it whatever is included in that term (**Bramantip, Dimaris**).

(2) Whatever is denied of a whole term may have wholly denied of it whatever is wholly included in that term (**Camenes**).

(3) Whatever is affirmed of the whole or part of a term may have partially denied of it whatever is wholly excluded from that term (**Fesapo, Fresison**).

Strengthened Syllogisms.

§ 322. A Strengthened Syllogism is one which assumes more in the premisses than is necessary to prove the conclusion.

The reader will observe that this is the case with **Bramantip** and **Fesapo** as compared with **Dimaris** and **Fresison** respectively. In **Bramantip** and **Fesapo** both premisses are universal. In **Dimaris** and **Fresison** one premiss is particular. Yet the conclusion of the stronger premisses is in each case identical with that of the weaker.

Among the two dozen legitimate combinations of mood and figure one third are strengthened syllogisms, there being two in each figure—

FIGURE	I.	AAI,	EAO.
FIGURE	II.	EAO,	AEO.
FIGURE	III.	AAI,	EAO.
FIGURE	IV.	AAI,	EAO.

It is only in the first and second figures that the syllogism with strengthened premiss coincides with the syllogism with weakened conclusion, which is otherwise known as a subaltern mood. In the third figure there are no subaltern moods, since it can prove only particulars. In the fourth figure AEO is a subaltern mood,

but it is not a strengthened syllogism[1]. If the major premiss were particular, we should have an illicit process of the major term; if the minor premiss were particular, we should have undistributed middle. The same evidence is here requisite for the particular as for the universal.

<div align="center">

Camenos.

All tomatos are fruits.	All P is M.
No fruits are roots.	No M is S.
∴ Some roots are not tomatos.	∴ Some S is not P.

</div>

[1] On the subject of Strengthened Syllogisms I am indebted to Keynes' *Formal Lo ic,* § 147, 1st edit.

CHAPTER XXVII.

Of the Syllogism with Three Figures.

§ 323. THE awkward fourth figure is no part of the logical system of Aristotle, but is said to have been added by Galen.

§ 324. In determining the number of figures we may look only at the position of the middle term in the premisses. There, it is clear, the middle term must either be subject in one and predicate in the other or else predicate in both or subject in both. We cannot tell the major from the minor premiss, unless we take account of the conclusion, which fixes the major and the minor term. Looked at from this point of view then the first and the fourth figures run into one, being that arrangement of terms in the premisses in which the middle term occupies a different position in one from what it does in the other.

§ 325. Aristotle's own way of viewing the matter seems rather to have been this. Either the middle term is really intermediate between the two extremes—

$$S\text{---}M\text{---}P. \qquad \begin{matrix} S\text{---}M \\ M\text{---}P \end{matrix}$$

or else the two extremes are outside of it

$$S\text{---}P\text{---}M. \qquad \begin{matrix} S\text{---}M \\ P\text{---}M \end{matrix}$$

or, lastly, they are inside of it

$$M\text{---}S\text{---}P. \qquad \begin{matrix} M\text{---}S \\ M\text{---}P. \end{matrix}$$

The fourth figure is got by inverting the order of the first—

$$P—M—S. \qquad \begin{array}{c} P—M \\ M—S. \end{array}$$

The clumsiness of the fourth figure is due just to this, that a term which is naturally subject is used as predicate. Instead of reasoning thus—

> All S is M
> All M is P (Figure I)
> ∴ All S is P

we reason thus—

> All S is M
> All M is P   (Figure IV)
> ∴ Some P is S.

§ 326. When the conclusion is set out of sight, the number of possible moods is the same as the number of combinations that can be made of the four things, A, E, I, O, taken two together, without restriction as to repetition. These are the following sixteen :—

AA	EA	IA	OA
AE	~~EE~~	IE	~~OE~~
AI	EI	~~II~~	~~OI~~
AO	~~EO~~	~~IO~~	~~OO~~.

The rules of syllogism relating to the premisses are fatal to seven of these, which reduces the valid moods to nine—

AA. AE. AI. AO. EA. EI. IA. IE. OA.

But, as the order of the premisses is now indifferent, four of these, namely, EA, IA, IE, OA are only repetitions

of preceding ones. Thus we are left with only five really different moods—

<p style="text-align:center">AA. AE. AI. AO. EI.</p>

§ 327. We will now put these moods into the first figure and draw from them all the conclusions of which they admit. Though the order of the premisses is of no moment, the distribution of the terms in them is of vital importance. We shall have therefore to recognise a difference corresponding to that between the first and fourth figures in the fourfold scheme.

Let it be premised that—

> when the extreme in the premiss that stands first is predicate in the conclusion, we are said to have a Direct Mood ;
>
> when the extreme in the premiss that stands second is predicate in the conclusion, we are said to have an Indirect Mood.

<p style="text-align:center">FIGURE I.</p>

<p style="text-align:center">Mood A A.</p>

All M is $\breve{P}$.	All $\bar{P}$ is M.
All $\bar{S}$ is M.	All M is $\breve{S}$.
∴ All S is $\breve{P}$ (**Barbara**).	∴ Some $\breve{S}$ is $\breve{P}$ (**Bramantip**).
or Some $\breve{S}$ is $\breve{P}$ (**Barbari**).	or Some $\breve{P}$ is $\breve{S}$ (**Barbari**).
or Some $\breve{P}$ is $\breve{S}$ (**Bramantip**).	

<p style="text-align:center">Mood A E.</p>

All M is $\breve{P}$.	All $\bar{P}$ is M.
No $\bar{S}$ is M.	No M is $\bar{S}$.
∴ Illicit Major.	∴ No S is $\bar{P}$ (**Camenes**).
	or Some $\breve{S}$ is not $\bar{P}$ (**Camenos**).
or Some $\breve{P}$ is not $\bar{S}$ (**Fesapo**).	or No $\bar{P}$ is $\bar{S}$ (**Celarent**).
	or Some $\breve{P}$ is not $\bar{S}$ (**Celaront**).

Mood A I.

All M is P̆.
Some S̆ is M.

∴ Some S̆ is P (**Darii**).
or Some P̆ is S̆ (**Dimaris**).

All P is M̆.
Some M̆ is S.

∴ Undistributed Middle.

Mood A O.

All M is P̈.
Some S̆ is not M.

∴ Illicit Major.
or Illicit Minor.

All P is M̆.
Some M̆ is not S.

∴ Undistributed Middle.

Mood E I.

No M is P̄.
Some S̆ is M.

∴ Some S̆ is not P (**Ferio**).
or Illicit Minor.

No P̄ is M.
Some M is S̆.

∴ Some S̆ is not P̄ (**Fresison**).
or Illicit Minor.

Of the above ten moods it will be seen that three are altogether invalid.

Of the remaining seven, one yields no less than four conclusions, one three, two yield two apiece, and the remaining three one each. This gives us a total of fourteen conclusions. But two of our syllogisms, namely **Barbari** (subaltern to **Barbara**) and **Bramantip** occur twice over. This reduces the whole number to twelve, of which half belong to the first and half to the fourth figure under the fourfold division.

§ 328. The application of the same method to the imperfect figures is a simpler matter, since we have now only one position of the middle term, and consequently also of the major and minor, to deal with.

Figure II.

Mood A A.

All P is M̆.

All S is M̆.

∴. Undistributed Middle.

Mood A E.

All $\overline{P}$ is M.

No $\overline{S}$ is M.

∴. No $\overset{..}{S}$ is $\overline{P}$ (**Camestres**).

or Some $\overset{\smile}{S}$ is not $\overline{P}$ (**Camestros**).

or No $\overline{P}$ is $\overline{S}$ (**Cesare**).

or Some $\overline{P}$ is not $\overline{S}$ (**Cesaro**).

Mood A I.

All P is M̆.

Some S is M̆.

∴. Undistributed Middle.

Mood A O.

All $\overline{P}$ is M.

Some $\overset{\smile}{S}$ is not M.

∴. Some $\overset{\smile}{S}$ is not $\overline{P}$ (**Baroco**).

or Illicit Minor.

No $\overline{P}$ is M.

Some $\overset{\smile}{S}$ is M.

∴. Some $\overset{\smile}{S}$ is not $\overline{P}$ (**Festino**).

or Illicit Minor.

Of the above five moods two are altogether invalid owing to the absence of a negative premiss. Of the remaining three AE yields four conclusions, while AO and EI yield one apiece. Thus we arrive at the six moods which are valid in the second figure.

Figure III.

§ 329.

Mood AA.

All M is P̆.

All M is S̆.

∴ Some S̆ is P̆ (**Darapti**).

or Some P̆ is S̆ (**Daraptis**).

Mood AE.

All M is P̆.

No M is S̄.

∴ Illicit Major.

or Some P̆ is not S̄ (**Felapton**).

Mood AI.

All M is P̆.

Some M is S̆.

∴ Some S̆ is P̆ (**Datisi**).

or Some P̆ is S̆ (**Disamis**).

Mood AO.

All M is P̆.

Some M is not S̄.

∴ Illicit Major.

or Some P̆ is not S̄ (**Bocardo**).

Mood EI.

No M is $\overline{P}$.

Some M is $\breve{S}$.

∴. Some $\breve{S}$ is not $\overline{P}$ (**Ferison**).

or Illicit Minor.

In this figure two moods are valid both directly and indirectly, the remaining three either directly or indirectly. This gives us in all seven syllogisms, one of which is not indicated by the mnemonic lines. We have called it in passing **Daraptis**, as being **Darapti** with its conclusion simply converted. It stands in the same relation to **Darapti** as **Disamis** does to **Datisi**, but has been overlooked owing to the accident of both premisses being the same kind of proposition. It spoils the symmetry of six valid syllogisms in each figure, which has perhaps been another reason for neglecting it. Under the threefold division of figure it falls into its natural place as the indirect mood to **Darapti**, but under the fourfold it presents itself as an uncomfortable interloper. It cannot be called a different mood from **Darapti**, since the propositions of which it is made up are the same in kind with that, and yet it is undoubtedly a different syllogism. Moreover if we convert the conclusion of the six moods that are valid in each of the four figures, we shall find that in every case except this the result, if it be legitimate, is provided in the mnemonic lines with a name of its own. For the names of the subaltern moods we may avail ourselves of the simple device already employed.

§ 330.

FIGURE I.		FIGURE II.	
Barbara	= Bramantip.	Cesare	= Camestres.
Celarent	= Camenes.	Camestres	= Cesare.
Darii	= Dimaris.	Festino	= Illicit Minor.
Ferio	= Illicit Minor.	Baroco	= Illicit Minor.
Barbari	= Bramantip.	Cesaro	= Camestros.
Celaront	= Camenos.	Camestros	= Cesaro.

FIGURE III.		FIGURE IV.	
Darapti	= Daraptis.	Bramantip	= Barbari.
Disamis	= Datisi.	Camenes	= Celarent.
Datisi	= Disamis.	Dimaris	= Darii.
Felapton	= Illicit Minor.	Fesapo	= Illicit Minor.
Bocardo	= Illicit Minor.	Fresison	= Illicit Minor.
Ferison	= Illicit Minor.	Camenos	= Celaront.

§ 331. Whenever the conclusion is O, the result of converting it is an illicit minor, except in the case of the subaltern moods, **Celaront** and **Camenos**, in which the O proposition is only a part of the full conclusion.

§ 332. The preceding list shows that under the fourfold division of figure the indirect moods of the first are to be found in the fourth and vice versâ, whereas the indirect moods of the second and third are to be found in those figures themselves.

CHAPTER XXVIII.

Of Reduction.

§ 333. REDUCTION may be taken as a general term for changing one form of reasoning into another.

The syllogism required to be reduced may be called the Reducend; that to which it conforms, when reduced, may be called the Reduct.

§ 334. The importance of reduction is confined to bringing moods in the imperfect figures into those of the first or perfect figure.

§ 335. The object of reduction, as thus conceived, is to extend the sanction of the canon of reasoning to syllogisms which do not obviously comply with it. Under the canon of reasoning, or *dictum de omni et nullo*, the essence of mediate inference consists in showing that a general principle applies to some special case or cases or a whole class of cases. The general principle is the major premiss; the assertion that something falls under it is the minor premiss. Hence the major premiss in a perfect syllogism must be universal, but may be negative; the minor premiss must be affirmative, but may be particular.

Now a good deal of our ordinary reasoning does not

conform to this type. If we adhere therefore to the canon of reasoning, it becomes necessary to show that all syllogistic reasoning may by a little manipulation be forced into compliance with it. This process is reduction in the commonly accepted sense of the term.

§ 336. Reduction is of two kinds—

(1) Direct or Ostensive.
(2) Indirect or *Per Impossibile.*

§ 337. The problem of direct or ostensive reduction is this—

Given any mood in one of the imperfect figures, how to alter the form of the premisses so as to arrive at the same conclusion in the perfect figure, or at one from which the old conclusion can be immediately inferred.

The alteration of the premisses is effected by means of immediate inference and, where necessary, of transposition.

§ 338. The problem of indirect reduction or *reductio per (deductionem ad) impossibile* is this—

Given any mood in one of the imperfect figures, to show by means of a syllogism in the perfect figure that its conclusion cannot be false.

Direct or Ostensive Reduction.

§ 339. In the usual form of direct reduction the only kind of immediate inference employed is conversion, either simple or by limitation.

The mnemonic lines, ' **Barbara, Celarent,**' &c. provide complete directions for the ostensive reduction of all

the moods of the second, third, and fourth figures to the first with the exception of **Baroco** and **Bocardo.** For the present we may take c in the body of a word to indicate that the syllogism of which that word is the name cannot be reduced ostensively by the ordinary methods. Leaving these two forms of syllogism then for subsequent treatment, we will deal at once with the remainder.

It will be observed that the names of the moods which are valid and useful in the first figure begin with the first four consonants in the alphabet, B, C, D, F. The initial consonant of any other figured mood indicates that the reduct will be that mood of the first figure which begins with the same letter. Thus the B of **Bramantip** shows that, when reduced, it will become **Barbara.**

Where m appears in the name of a reducend, it stands for *mutatio* or metathesis of the premisses. We may interpret it to mean—Make the major the minor and the minor the major.

When s follows one of the premisses of a reducend, it indicates that the proposition to which it is appended must be simply converted ; when it follows the conclusion, as in **Disamis**, it indicates that the conclusion arrived at in the first figure is not identical in form with the original conclusion, but capable of being inferred from it by simple conversion. Hence s in the middle of a name indicates something to be done to the original premiss, while s at the end indicates something to be done to the new conclusion.

p stands for conversion *per accidens.* What has just

been said of the difference between s in the middle and
at the end of a word applies also to p.

The letters l, n, r, t are meaningless.

FIGURE II.

§ 340.   **Cesare.**   **Celarent.**

E.	No P is M.			No M is P.	E.
A.	All S is M.	$\Big\}=\Big\{$		All S is M.	A.
E.	∴ No S is P.			∴ No S is P.	E.

Camestres.   **Celarent.**

A.	All P is M.			No M is S.	E.
E.	No S is M.	$\Big\}=\Big\{$		All P is M.	A.
E.	∴ No S is P.			∴ No P is S.	E.
				∴ No S is P.	

Festino.   **Ferio.**

E.	No P is M.			No M is P.	E.
I.	Some S is M.	$\Big\}=\Big\{$		Some S is M.	I.
O.	∴ Some S is not P.			∴ Some S is not P.	O.

[Baroco.]

FIGURE III.

§ 341.   **Darapti.**   **Darii.**

A.	All M is P.			All M is P.	A.
A.	All M is S.	$\Big\}=\Big\{$		Some S is M.	I.
I.	∴ Some S is P.			∴ Some S is P.	I.

Disamis.   **Darii.**

I.	Some M is P.			All M is S.	A.
A.	All M is S.	$\Big\}=\Big\{$		Some P is M.	I.
I.	∴ Some S is P.			∴ Some P is S.	I.
				∴ Some S is P.	

	Datisi.			**Darii.**	
A.	All M is P.	⎞	⎛	All M is P.	A.
I.	Some M is S.	⎬ = ⎨	Some S is M.	I.
I.	∴ Some S is P.	⎠	⎝	∴ Some S is P.	I.

	Felapton.			**Ferio.**	
E.	No M is P.	⎞	⎛	No M is P.	E.
A.	All M is S.	⎬ = ⎨	Some S is M.	I.
O.	∴ Some S is not P.	⎠	⎝	∴ Some S is not P.	O.

[Bocardo.]

	Ferison.			**Ferio.**
	No M is P.	⎞	⎛	No M is P.
	Some M is S.	⎬ = ⎨	Some S is M.
	∴ Some S is not P.	⎠	⎝	∴ Some S is not P.

FIGURE IV.

§ 342.

	Bramantip.			**Barbara.**	
A.	All P is M.	⎞	⎛	All M is S.	A.
A.	All M is S.	⎬ = ⎨	All P is M.	A.
I.	∴ Some S is P.	⎠	⎝	∴ All P is S.	A.
				∴ Some S is P.	

	Camenes.			**Celarent.**	
A.	All P is M.	⎞	⎛	No M is S.	E.
E.	No M is S.	⎬ = ⎨	All P is M.	A.
E.	∴ No S is P.	⎠	⎝	∴ No P is S.	E.
				∴ No S is P.	

	Dimaris.			**Darii.**	
I.	Some P is M.	⎞	⎛	All M is S.	A.
A.	All M is S.	⎬ = ⎨	Some P is M.	I.
I.	∴ Some S is P.	⎠	⎝	∴ Some P is S.	I.
				∴ Some S is P.	

	Fesapo.			**Ferio.**	
E.	No P is M.	⎫ ⎧	No M is P.	E.
A.	All M is S.	⎬ = ⎨	Some S is M.	I.
O.	∴ Some S is not P.	⎭ ⎩	∴ Some S is not P.	O.

	Fresison.			**Ferio.**	
E.	No P is M.	⎫ ⎧	No M is P.	E.
I.	Some M is S.	⎬ = ⎨	Some S is M.	I.
O.	∴ Some S is not P.	⎭ ⎩	∴ Some S is not P.	O.

Reduction of the Subaltern Moods.

§ 343. The mnemonic lines do not provide for the reduction of the three subaltern moods in the imperfect figures **Cesaro**, **Camestros**, and **Camenos**. The first of these runs easily into **Celarent**.

	Cesaro.			**Celaront.**	
E.	No P is M.	⎫ ⎧	No M is P.	E.
A.	All S is M.	⎬ = ⎨	All S is M.	A.
O.	∴ Some S is not P.	⎭ ⎩	∴ Some S is not P.	O.

The name here may be considered also to serve as a symbol for reduction, if it be borne in mind that the reduct is **Celaront**. But in the case of **Camestros** and **Camenos** the reduct is not **Celaront** but **Celarent**. We have first to draw the full conclusion and then submit it to simple conversion followed by subalternation. There is nothing in the name to indicate this last step.

	Camestros.			**Celarent.**	
A.	All P is M.	⎫ ⎧	No M is S.	E.
E.	No S is M.	⎬ = ⎨	All P is M.	A.
O.	∴ Some S is not P.	⎭ ⎩	∴ No P is S.	E.
			∴ No S is P.	
			∴ Some S is not P.	

	Camenos.			Celarent.	
A.	All P is M.			No M is S.	E.
E.	No M is S.			All P is M.	A.
O.	∴ Some S is not P.			∴ No P is S.	E.
				∴ No S is P.	
				∴ Some S is not P.	

As simple conversion followed by subalternation is the same thing as conversion *per accidens,* we may call these two moods for purposes of reduction **Camestrop** and **Camenop.** Their names then will completely tally with those in the mnemonic lines.

Reduction by Negation.

§ 344. We now return to **Baroco** and **Bocardo,** which we left over for separate treatment.

The reason why these two moods cannot be reduced ostensively by the aid merely of conversion or transposition or both becomes plain on an inspection of them. In both it is necessary, if we are to obtain the first figure, that the position of the middle term should be changed in one premiss. But the premisses of both consist of A and O propositions, of which A admits only of conversion by limitation, the effect of which would be to produce two particular premisses, while O does not admit of conversion at all.

It is clear then that the O proposition must cease to be O before we can get any further. Here obversion comes to our aid; while conversion by negation enables us to convert the A proposition without loss of quantity.

(Baroco) Fanobo.   **Ferio.**

A.	All P is M.	No not-M is P. E.
O.	Some S is not M.	Some S is not-M. I.
O.	∴ Some S is not P.	∴ Some S is not P. O.

$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} = \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$

A.	All honest men are truth-ful.	No untruthful men are honest. E.
O.	Some clever men are not truthful.	Some clever men are untruthful. I.
O.	∴ Some clever men are not honest.	∴ Some clever men are not honest. O.

(Bocardo) Donamon.   **Darii.**

O.	Some M is not P.	All M is S. A.
A.	All M is S.	Some not-P is M. I.
O.	∴ Some S is not P.	∴ Some not-P is S. I.
		∴ Some S is not-P.
		∴ Some S is not P.

§ 345. In the new symbols **Fanobo** and **Donamon** b is employed as a sign of obversion (since o cannot be); n signifies conversion by negation. In Donamon the first n stands for a process which resolves itself into obversion followed by simple conversion, the second for one which resolves itself into simple conversion followed by obversion, according to the extended meaning which we have given to the term 'conversion by negation.' The student who finds himself perplexed by this difference may have recourse to the cacophonous names **Dobsamosb** and **Fabsobo**, in which the steps of simple immediate inference replace the compound result.

Extension of Reduction by Negation.

§ 346. The method of reduction by negation, though necessary only for the direct reduction of **Baroco** and **Bocardo**, will be found on trial to be applicable to any mood in the imperfect figures. As this process is a good logical exercise, we shall here present the student with a version of the mnemonic lines adapted for this purpose. They will furnish him with a test which he can apply to his own work.

> **Barbara, Celarent, Darii, Ferioque** prioris.
> **Bēnāreb, Cănĕbe, Dĕnĭlob, Fānōbŏ** secundae ;
> **Cēbamnes, Bămĕnenque** modi duo prima reponunt
> nomina, queis **Bĕnărob, Cănĕbō** sunt adjicienda,
> necnon **Cēbamnop, Bămĕnont,** si plena requiris.
> Tertia **Fampābin, Dēnāmon,** nec vice factâ
> **Fābāpib, Fmĭsăbin, Fābīsĭb** ; itemque **Dĕbāpob**
> **Dōnāmon, Dēbīsob** habet.   Dat quarta **Cămābin**
> **Cānanbinp, Cănĕnē, Fĭmăbin,** quibus adde **Dĕnāpob**
> cum **Dĕnĭsob** ; **Bămĕben** finem facient **Cănĕnōque,**
> si modo jam cumulum **Bamebont** licet adjiciamus.

n here stands for conversion by negation, whether by obversion-conversion or by conversion-obversion ; b for obversion ; t for subalternation ; l and r are meaningless ; the rest of the letters have the same signification as in the original lines.

Which form of conversion by negation n stands for is determined by the proposition which has to be arrived at.   Thus in **Canebe** A has to be converted by negation

and become the E of **Celarent**. It is therefore obversion-conversion that is required, since the other process would reduce A to O.

The above lines make provision for the reduction of the subaltern moods as well as of their originals.

In all cases in which both premisses are universal, with the exception of **Fesapo**, the mood admits of being reduced by negation in two ways, either as it stands or by transposition of the premisses. In the case of **Fesapo** the transposition of the premisses leads either to a negative minor in the first figure or to the fallacy of four terms.

In the second figure **Cebamnes** and **Bamenen** with their subalterns **Cebamnop** and **Bamenont** result from transposing the premisses of **Cesare** and **Camestres**.

In the third figure **Fampabin** and **Denamon** are the syllogisms which arise from transposing the premisses of **Darapti** and **Felapton**. The rest of the moods follow the old order of the lines.

In the fourth figure **Camabin**, which results from transposing the premisses of **Bramantip**, is an alternative for **Cananbinp**. **Bameben** stands in the same relation to **Canene**, being got like it from **Camenes**.

The subaltern moods display their relationship to their originals by their names. Thus **Bamebont** is subaltern to **Bameben**.

It should be mentioned that the reduct of **Benarob** is **Barbari**, but of **Bamenont Barbara**; of **Canebo Celaront**, but of **Bamebont Barbara**. Even this however is implied

by the symbols, since neither p nor t could refer to any-thing but a universal proposition.

§ 347. To show how reduction by negation may be employed in argument we will take a concrete instance of **Canebe.**

Canebe (Camestres).

All things of which we have a perfect idea are perceptions.

A substance is not a perception.

∴ A substance is not a thing of which we have a perfect idea.

Celarent.

No not-perception is a thing of which we have a perfect idea.

A substance is a not-perception.

∴ A substance is not a thing of which we have a perfect idea.

We may go on, if we please, to reduce this to **Barbara** by obverting the major premiss, so as to obtain the contrapositive of the original.

Barbara.

All not-perceptions are things of which we have an imperfect idea.

A substance is a not-perception.

∴ A substance is a thing of which we have an imperfect idea.

∴ A substance is not a thing of which we have a perfect idea.

Indirect Reduction.

§ 348. The impossibility of reducing **Baroco** and **Bocardo** ostensibly without the employment of negative terms was what led to the adoption of the indirect method.

Let us apply it to **Baroco.**

Baroco.

All P is M.	All fishes are oviparous.
Some S is not M.	Some marine animals are not oviparous.
Some S is not P.	∴ Some marine animals are not fishes.

If possible, let this conclusion be false.

Then its contradictory must be true.

∴ It is true that—All S is P. All marine animals are fishes.

Combining this as minor with the original major, we obtain premisses in the first figure—

Barbara.

All P is M.	All fishes are oviparous.
All S is P.	All marine animals are fishes—

which necessitate the conclusion,

All S is M.	All marine animals are oviparous.

But this conclusion conflicts with the original minor, being its contradictory,

∴ One of them must be false.

But the original minor is given as true.

∴ The new conclusion is false.

∴ The syllogism in which it is drawn must be wrong
either in form or in matter.

But it is in the first figure, to which the Canon of
Reasoning applies.

∴ It is not wrong in form.

∴ It is wrong in matter, i. e. one of the premisses must
be false.

Now the major premiss is given as true, being the
same as in the original syllogism.

∴ The minor premiss is false.

I. e. It is false that—

All S is P.   All marine animals are fishes.

But this proposition has already been shown to be
true.

∴ It is both true and false.

I. e. We are involved in a contradiction.

∴ The supposition which leads to it must be abandoned.
And this supposition is that the original conclusion
is false.

∴ The original conclusion is true.

The essence of the reduction consists in the appeal
to the canon of reasoning. If we stopped short at
showing that the new conclusion contradicts the old
premiss, it would be open to an objector to reply that
the fault lay with our new syllogism.

The premisses of **Baroco** are given as true, the

question at issue not being about their truth as propositions, but merely as to the conclusion drawn from them in the second figure, which does not obviously conform to the canon. We show that the truth of the canon indirectly implies the truth of this conclusion.

It is usual to place the two syllogisms side by side thus—

Baroco.	**Barbara.**
All P is M.	All P is M.
Some S is not M.	All S is P.
∴ Some S is not P.	∴ All S is M.

But the student must not imagine that this constitutes reduction without his appending the reasoning in which the process really consists.

In **Bocardo** the two syllogisms will stand thus—

Bocardo.	**Barbara.**
Some M is not P.	All S is P.
All M is S.	All M is S.
∴ Some S is not P.	∴ All M is P.

The lines which we have drawn indicate the propositions which conflict with one another.

The names **Baroco** and **Bocardo** were invented with a view to this process. The initial consonant tells that the indirect reduct is **Barbara.** The c is a direction to substitute the contradictory of the old conclusion for the proposition after which it is placed, which in both these cases is o.

§ 349. A direct proof gives a reason why a thing is as it is.

An indirect proof gives a reason why a thing cannot be otherwise than as it is.

Indirect proof then is inferior to direct proof, inasmuch as it is negative, not positive; it only informs us *that* a thing is, not *why* it is. But, like other things of less value, it is commoner. Indirect reduction is a form of indirect proof.

In indirect proof we may take any true proposition, and combine it with the contradictory of the one which is called in question, in such a way as to form a syllogism; in indirect reduction we are limited to one of the premisses of an original syllogism.

§ 350. On what principle does indirect reduction rest? It rests on the rules relating to the material quality of the propositions in a syllogism (§ 295).

The first of these asserts that—

> If both premisses be true, the conclusion must be true.

The second, which is the contrapositive of this, adds that—

> If the conclusion be false, one or both premisses must be false.

From this last it is an easy deduction that—

> If the conclusion be false and one premiss be true, the other premiss must be false.

In the new syllogism which we frame the conclusion

is false, because it conflicts with one of the old premisses, which are not called in question ; but one of its premisses is true, being derived from the old syllogism ; therefore the other premiss is false. Now the other premiss is the contradictory of the original conclusion and has already been shown to be true. Therefore we are involved in a contradiction. Therefore we must abandon the hypothesis which leads to it.

Extension of Indirect Reduction.

§ 351. It so happens that the two combinations of mood and figure for which this process was originally invented are just the two to which it is least applicable.

In **Baroco** the major premiss is combined with the contradictory of the conclusion to prove the falsity of the minor.

In **Bocardo** the minor premiss is combined with the contradictory of the conclusion to prove the falsity of the major.

Now could we effect the indirect reduction of **Baroco** by retaining the minor premiss or that of **Bocardo** by retaining the major? The answer is in both cases ' No,' and for the same reason, namely, that O cannot be used as a premiss in the first figure at all. But, if we take any other mood in the imperfect figures, we shall find that it can be indirectly reduced to the first figure either by retaining the major or by retaining the minor premiss. The reader can verify this assertion for himself. Instead of encumbering our pages by exhibiting the process in each case

we will follow the convenient example set by Peter of Spain and his predecessors. The following mnemonic lines contain directions for reducing all the moods of the imperfect figures to the first, whether by way of the major or of the minor premiss.

> Barbara, Celarent, Darii, Ferioque prioris;
> Fēlăcĕ, Dārēcē, Cēlĭcŏ, Bărōcŏ secundae,
> sedem si retinet major; sed, si minor, inde
> tertia Cācănĭt, Cĭcărī, Fācĭnĭ, Bĕcănot,
> Bōcardō, Dēcĭlon habet; quarta insuper addit
> Fampacsī, Dănĕces, necnon Fimsācĭ, Cĕnācop,
> praeterea detur salva majore Cĕnicos.
> Adde subalternos Cēlācŏ, Bălēcŏ secundae;
> Bānēcos quartae tribuatur rite figurae.

> Altera si praemissa manet, novus incipit ordo.
> Namque secunda dabit Decsāmĕ, minore retentâ,
> Facsēmē Dĕcĭsosque modi reddantur eodem.
> Tertia praebebit salva majore Cămacsit,
> Fīmacsī, pariterque Cămicsī, deinde Cēsācop,
> queis, si vis totum, licet annumerare Cĕsīcos.
> Quartaque Cācānip profert Facsemsĕ, Cĭcanis,
> Bēcānop Dĕcĭlos, retinet si sede minorem.
> Adde subalternos etiam Decpamŏ secundae
> Facpēmēque super; Facpemsō quarta requirit.

The symbols are constructed on the model of **Baroco** and **Bocardo,** the initial consonant indicating the mood of the indirect reduct and the c being put after the premiss which is to be replaced by the contradictory of the

conclusion. m means that the retained premiss will have
to shift its place in the reduct, if it was major becoming
minor, and, if minor, major. As in the symbols for direct
reduction, s and p in the body of a word signify something
to be done to the old premiss, at the end something to be
done to the new conclusion. l, n, r are meaningless, but t
now signifies that the subaltern mood may be employed
in the reduct.   Thus **Camacsit (Darapti)** falls into
Celaront.   The full mood, **Celarent**, would yield the
contrary, not the contradictory of the original minor.
The contrary however will do equally well, being as
incompatible with the *truth* of the original proposition as
the contradictory.   The three subaltern moods of the
imperfect figures should be reduced to the subalterns of
the first.

Converse Reduction.

§ 352. By Converse Reduction is meant bringing
a syllogism in the perfect figure into one of the imperfect
ones.   The process is only valuable as an exercise.

§ 353. As all the moods of the imperfect figures can be
reduced into those of the first, it might on first hearing be
assumed that all the moods of the first figure can be
reduced into those of the rest.   But the relation is not
quite reciprocal.   It is one of the perfections of the first
figure that, except in the subaltern moods, it never
assumes more in the premisses than what is necessary to
prove the conclusion, i. e. it never has a term superfluously
distributed.   This characteristic is shared with it by the
second figure, but not by the other two.   Thus **Darapti**

takes two universal propositions to prove the same conclusion which in **Disamis** or **Datisi** is established with the aid of one; and the same is the case with **Bramantip** as compared with **Dimaris**. Now a weaker proposition in the same matter may always be substituted for a stronger, but never a stronger for a weaker. Hence it follows that we cannot reduce a mood in the first figure to one in the third or fourth, if the latter differs from the former by having one of its premisses strengthened.

§ 354. The reduction of **Barbara** to **Bramantip** stands apart from the rest, as being the only case in which the conclusion of the reduct is a different kind of proposition from that of the reducend. When we say that **Barbara** is reducible to the fourth figure, we mean that we can substitute another mood for it in that figure. We cannot take the mood A A A itself and express it in the fourth figure, the A conclusion being the exclusive prerogative of the first.

Converse Use of Indirect Reduction.

§ 355. It has not been much noticed that indirect reduction is a reciprocal process. Let us give a formal proof of this.

Proof that Indirect Reduction is a reciprocal process.

If we take any syllogism and, while retaining one of its premisses, substitute the contradictory of its conclusion for the other premiss, the conclusion of this new syllogism will be the contradictory of that other premiss. If then, retaining the same premiss as before, we repeat the same

process on the new syllogism, we must get back to the original one. For one premiss is unchanged, and the other is the contradictory of the contradictory of the original, that is, it is the original premiss itself.

Felace (Cesare).	**Ferio.**	**Felace.**
No P is M.	No P is M.	No P is M.
All S is M.	Some S is P.	All S is M.
∴ No S is P.	∴ Some S is not M.	∴ No S is P.

Here the major premiss is the same throughout. By combining with it in the first reduct the contradictory of the original conclusion, we obtain as a new conclusion the contradictory of the original minor. But the contradictory of this is to become the next minor, that is, we must revert to the original minor itself.

§ 356. We have an apparent exception to reciprocity whenever we are compelled to reduce the quantity of a proposition. In such a case the next reduct has to sink to a lower level. An example will make this clearer.

Barbara.	**Festino.**	**Darii.**
All M is P.	No S is P.	All M is P.
All S is M.	Some M is S.	Some S is M.
∴ All S is P.	∴ Some M is not P.	∴ Some S is P.
∴ Some S is P.		

CHAPTER XXIX.

Of Complex Syllogisms.

§ 357. A Complex Syllogism is one which is composed, wholly or in part, of complex propositions.

§ 358. Though there are only two kinds of complex proposition, there are three varieties of complex syllogism. For we may have

> (1) a syllogism in which the only complex premiss is conjunctive,
>
> (2) a syllogism in which the only complex premiss is disjunctive.
>
> (3) a syllogism in which one premiss is conjunctive and the other disjunctive.

```
                        Syllogism
                            |
        _____
        |                                       |
      Simple                                 Complex
   (Categorical)                          (Conditional)
                                               |
        _____
        |                       |                          |
   Conjunctive            Disjunctive                   Dilemma¹.
  (Hypothetical)
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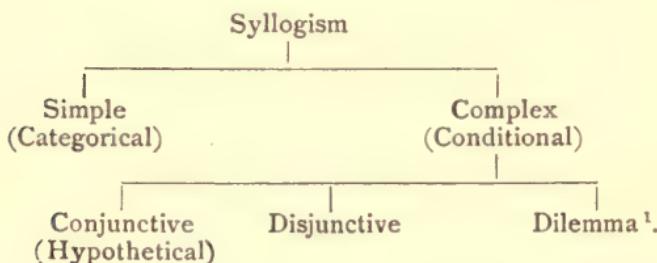
¹ The Conjunctive Syllogism is called by Cicero (*De Inv.* I, § 45) *simplex conclusio*, the Disjunctive *enumeratio*, and the dilemma *complexio*.   For the last cp. Asconius on *Verr. Div.* § 45.

The Conjunctive Syllogism.

§ 359. A Conjunctive Syllogism is one in which the only complex premiss is conjunctive.

The conclusion may be said to follow the weaker part, if we consider a simple proposition as being weaker than a conjunctive. Therefore

> when both premisses are conjunctive, the conclusion will be conjunctive;
>
> when one premiss is simple, the conclusion will be simple.

This gives us two kinds of conjunctive syllogism—

> (1) The Wholly Conjunctive Syllogism.
> (2) The Partly Conjunctive Syllogism.

The Wholly Conjunctive Syllogism.

§ 360. Since every conjunctive proposition may be read as a simple one, every syllogism also which is composed wholly of such propositions may be read as a simple syllogism. Wholly conjunctive syllogisms therefore do not differ essentially from simple ones. They may be constructed in every mood and figure. A couple of instances will suffice.

$$\text{Let A is B} = \text{P.}$$
$$\text{C is D} = \text{M.}$$
$$\text{E is F} = \text{S.}$$

Cesare.

If A is B, C is never D.   If a man is rash, he is never rational.
If E is F, C is always D.   If a man is brave, he is always rational.
∴. If E is F, A is never B.   ∴. If a man is brave, he is never rash.

Disamis.

If C is D, A is sometimes B.	If she goes, I sometimes go.
If C is D, E is always F.	If she goes, he always goes.
∴ If E is F, A is sometimes B.	∴ If he goes, I sometimes go.

The second of these instances may be read as a simple syllogism thus—

<div align="center">

Disamis.
Some CD is AB.
All CD is EF.
∴ Some EF is AB.

</div>

The Partly Conjunctive Syllogism.

§ 361. It is this kind which is usually meant when the conjunctive or hypothetical syllogism is spoken of.

§ 362. Of the two premisses, one conjunctive and the other simple, the conjunctive is considered to be the major, and the simple premiss the minor. For the conjunctive premiss lays down a certain relation to hold true universally between two propositions as a matter of theory, which is applied in the minor to a matter of fact.

§ 363. Taking a conjunctive proposition as a major premiss[1], there are four simple minors possible. For we may either assert or deny the antecedent or the consequent of the conjunctive.

Constructive Mood[2].	*Destructive Mood.*
(1) If A is B, C is D.	(2) If A is B, C is D.
A is B.	C is not D.
∴ C is D.	∴ A is not B.

[1] The major premiss in this form of argument was called by the Stoics the λῆμμα, the minor the πρόσληψις, and the conclusion the ἐπιφορά. Diog. Laert. VII, § 76.

[2] Τρόπος δέ ἐστιν, οἱονεὶ σχῆμα λόγου· οἷον ὁ τοιοῦτος, Εἰ τὸ α΄, τὸ β΄. Ἀλλὰ μὴν τὸ πρῶτον· τὸ ἄρα δεύτερον. *Ibid.*

Assertion of the Consequent.	*Denial of the Antecedent.*
(3) If A is B, C is D.	(4) If A is B, C is D.
C is D.	A is not B.
No conclusion.	No conclusion.

The constructive mood or assertion of the antecedent is also known as the *modus ponens*.

The destructive mood or denial of the consequent is also known as the *modus tollens*.

§ 364. From a consideration of the above four cases we elicit the following

Canon of the Conjunctive Syllogism.

To assert the antecedent is to assert the consequent and to deny the consequent is to deny the antecedent[1], but from asserting the consequent or denying the antecedent no conclusion follows.

§ 365. When we take as a minor ' C is D ' (3), we can get no universal conclusion. For though A being B is declared to involve as a consequence C being D, yet it is possible for C to be D under other circumstances or from other causes. Granting the truth of the proposition ' If the sky falls, we shall catch larks,' it does not follow that there are no other conditions under which this result can be attained.

§ 366. Again, when we take as a minor ' A is not B ' (4), it is clear that we get no conclusion at all. For

---

[1] Cic. *Fin.* IV, § 55   ita fit illa conclusio non solum vera, sed ita perspicua, ut dialectici ne rationem quidem reddi putent oportere : si illud hoc ; non autem hoc : igitur ne illud quidem.'

to say that C is D whenever A is B gives us no right
to deny that C can be D in the absence of that condition.
What we have predicated has been merely inclusion of the
case AB in the case CD.
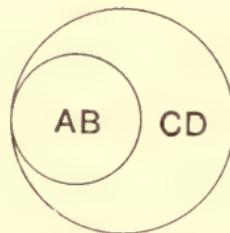
'If I get run over by an omnibus I shall be killed.'
Granted: but, if I escape that danger, does it follow that
I shall not be killed in some other way?

§ 367. There is a case however in which we can
obtain a conclusion both by asserting the consequent
and by denying the antecedent of a conjunctive proposi-
tion. It is when the relation predicated between the
antecedent and the consequent is not that of inclusion,
but of coincidence, where in fact the conjunctive pro-
position conforms to the type U. For example—

Assertion of the Consequent.

If you repent, then always and only are you forgiven.
You are forgiven.

∴ You repent.

Denial of the Antecedent.

If you repent, then always and only are you forgiven.
You do not repent.

∴ You are not forgiven.

The antecedent is here given as the sole, and at the same time all-sufficing, condition of the antecedent. If it were not all-sufficing, repentance could not be inferred from forgiveness, for it might still be the case that some who repent are not forgiven; if it were not the sole condition, the absence of forgiveness could not be inferred from the absence of repentance, for it might still be the case that some who do not repent are forgiven.

§ 368. The partly conjunctive syllogism may be exactly defined as follows—

A complex syllogism which has for its major premiss a conjunctive proposition, of which in the minor premiss the antecedent is asserted or the consequent denied.

Of the Reduction of the Partly Conjunctive Syllogism.

§ 369. Such syllogisms as those just treated of have a major and a middle term visible to the eye, but appear to be destitute of a minor. The missing minor however is latent in the transition from hypothesis to fact. When we say 'A is B,' we mean (rightly or wrongly) 'As a matter of fact A is B' or 'The actual state of the case is that A is B.' The insertion therefore of some such expression as 'The case in hand' or 'This case' is all that is wanted to complete the form of the syllogism. When reduced in this manner to the simple type of argument, it will be found that the constructive conjunctive conforms to the first figure, and the destructive conjunctive to the second.

Constructive Mood.	**Barbara.**
If A is B, C is D.	All AB is CD.
A is B.	This is AB.
∴ C is D.	∴ This is CD

Destructive Mood.	**Camestres.**
If A is B, C is D.	All AB is CD.
C is not D.	This is not CD.
∴ A is not B.	∴ This is not AB.

§ 370. As the partly conjunctive syllogism is thus reducible to the simple form, we may suspect *a priori* that violations of its laws will correspond with violations of the laws of simple syllogism. On experiment we find this to be the case.

Assertion of the Consequent.   Undistributed Middle.

$$\left.\begin{array}{l}\text{If A is B, C is D.}\\\text{C is D.}\end{array}\right\} = \left\{\begin{array}{l}\text{All AB is C\v{D}.}\\\text{This   is C\v{D}.}\end{array}\right.$$

Denial of the Antecedent.   *Illicit Major.*

$$\left.\begin{array}{ll}\text{If A is B, C is D.}\\\text{A is not B.}\\\therefore\qquad\text{C is not D.}\end{array}\right\} = \left\{\begin{array}{l}\text{All AB is C\v{D}.}\\\text{This is not AB.}\\\therefore\text{ This is not C\=D.}\end{array}\right.$$

§ 371. If we judge the foregoing arguments by the special rules of the figures to which they belong, we see that the assertion of the consequent involves two affirmative premises in the second figure, and the denial of the antecedent a negative minor in the first.

By making the consequent of the major premiss negative we might avoid breaking the rule of the second figure, but this would only land us in the fallacy of two negative premisses instead.

$$\left.\begin{array}{l}\text{If A is B, C is not D.}\\\text{C is not D.}\end{array}\right\} = \left\{\begin{array}{l}\text{No AB is   CD.}\\\text{This is not CD.}\end{array}\right.$$

Similarly by making the antecedent of the major premiss negative we might avoid breaking the rule of the first figure, but this would only land us in the fallacy of four terms instead.

If A is not B, C is D. $\left.\right\}$ = $\left\{\right.$ All not-AB is CD.

A is B. $\qquad$ This $\qquad$ is AB.

§ 372. We have seen that the partly conjunctive syllogism falls, when constructive, into the first figure and, when destructive, into the second. We shall now see that its canon is in harmony with this mixed character.

The first clause of the canon of the conjunctive syllogism—'To assert the antecedent is to assert the consequent'—corresponds with the *Dictum de Omni*. For whereas something (namely C being D) is affirmed in the major of all conceivable cases of A being B, the same is affirmed in the conclusion of something which is included therein, namely, 'this case' or 'some cases' or even 'all actual cases.'

The second clause—'to deny the consequent is to deny the antecedent'—corresponds with the *Dictum de Diverso* (§ 317). For whereas in the major all conceivable cases of A being B are included in C being D, in the minor 'this case' or 'some cases' or even 'all actual cases of C being D' are excluded from the same term.

§ 373. The special characteristic of the partly conjunctive syllogism lies in the transition from hypothesis to fact. We might lay down as the appropriate axiom of this form of argument, that 'What is true in the abstract is true in the concrete' or 'What is true in theory is also true in fact,' a proposition which is apt to be neglected or denied. But this does not vitally differentiate it from the ordinary syllogism. For though in the latter we think rather of the transition from a

general truth to a particular application of it, yet at bottom a general truth is nothing but an hypothesis resting upon a slender basis of observed fact. We believe that all men die ; but how few have we seen do so ! How few perhaps are the men that have yet been compared with the men that are yet to be ! Yet of all of them we assert that they are mortal. This is only an hypothesis which nature is always verifying.

CHAPTER XXXI.

Of the Disjunctive Syllogism.

§ 374. A Disjunctive Syllogism is one in which the only complex premiss is disjunctive.

The disjunctive premiss is regarded as the major, since it lays down an hypothesis which is applied to fact in the minor.

§ 375. Taking a disjunctive proposition with only two alternatives as the major premiss, there are four simple minors possible. For we may assert or deny either of the alternatives.

Constructive Moods.

(1) Either A is B or C is D.	(2) Either A is B or C is D.
A is not B.	C is not D.
∴ C is D.	∴ A is B.

Either death is annihilation or we are immortal.	Either the water is shallow or the boys will be drowned.
Death is not annihilation.	The boys are not drowned.
∴ We are immortal.	∴ The water is shallow.

Destructive Moods.

(3) Either A is B or C is D.	(4) Either A is B or C is D.
A is B.	C is D.
No conclusion.	No conclusion.

A successful student must be either clever or industrious.	Either your play is bad or your luck is.
He is clever.	Your luck is bad.—
No conclusion.	No conclusion.

The constructive moods are also known as *modi tollendo ponentes.*

The destructive moods are also known as *modi ponendo tollentes.*

§ 376. It is only the constructive moods that are formally valid. The validity of the two destructive moods is contingent upon the kind of alternatives selected.

If these are such as necessarily to exclude one another, the conclusion will hold, but not otherwise.

They are of course mutually exclusive, whenever they embody the result of a correct logical division, as ' Triangles are either equilateral, isosceles, or scalene.' Here, if we affirm one of the members, we are justified in denying the rest. When the major thus contains the dividing members of a genus, it may more fitly be symbolized under the formula ' A is either B or C.' But as this admits of being read in the shape ' Either A is B or A is C,' we retain the wider expression which includes it.

Any knowledge however which we have of the fact that the alternatives selected in the major are incompatible must come to us from material sources ; unless

indeed we have confined ourselves to a pair of contradictory terms (A is either B or not-B). There can be nothing in the form of the expression to imply the incompatibility of the alternatives, since the same form is employed when the alternatives are palpably compatible. When a man says 'There will be rain either to-day or to-morrow,' we do not consider him confuted by the result, if it rains both days.

§ 377. There is no limit to the number of members in the disjunctive major. But if there be more than two alternatives, the conclusion will itself be disjunctive, unless the minor denies all but one.

A is either B or C or D.	A is either B or C or D.
A is not D.	A is neither C nor D.
∴ A is either B or C.	∴ A is B.

Virtue is either a state or a capacity or an affection of the soul.	Virtue is either a state or a capacity or an affection of the soul.
Virtue is not an affection of the soul.	Virtue is neither a capacity nor an affection of the soul.
∴ Virtue is either a state or a capacity.	∴ Virtue is a state.

§ 378. If we take the disjunctive proposition as limited to two alternatives, we may lay down the following

Canon of the Disjunctive Syllogism.

To deny one alternative is to assert the other, but from asserting either nothing follows.

If we take into account a plurality of alternatives, we must extend our canon thus—

To deny one member is to affirm the rest, either simply or disjunctively; but from affirming any member nothing follows.

§ 379. The disjunctive syllogism with two alternatives may be exactly defined as follows—

A complex syllogism which has for its major premiss a disjunctive proposition, either the antecedent or consequent of which is in the minor premiss simply denied.

CHAPTER XXXII.

Of the Dilemma.

§ 380. THE dilemma belongs to the third kind of complex syllogism in which one premiss is conjunctive and the other disjunctive. It would be possible however to combine two such premisses into a valid syllogism which should not be a dilemma, e.g.

> If C is D, E is F.
> Either A is B or C is D.
> ∴ Either A is B or E is F.

We shall therefore have to define precisely the relation in which the two propositions which form the premisses stand to one another. This however will be done to more advantage at the end of the discussion than at the beginning.

§ 381. It will facilitate the comprehension of the dilemma, if the following four points are borne in mind.

The first relates to the major premiss;
the second to the minor premiss;
the third to the conclusion;
the fourth to the canon with which the dilemma complies.

(1) The major premiss is a conjunctive proposition with more than one antecedent or more than one consequent or more than one of both.

(2) The minor premiss must be disjunctive.

(3) If there be only one antecedent or only one consequent, the conclusion will be a simple proposition; if there be more than one of both, the conclusion will itself be disjunctive.

(4) The dilemma complies with the canon of the conjunctive syllogism. The two things therefore which are permissible in the minor are to affirm the antecedent or deny the consequent.

§ 382. What is it then that differentiates the dilemma from any other conjunctive syllogism? It is the presence of a disjunctive minor, which is rendered possible by the plurality of antecedents or consequents in the major. It is this which constitutes the essence of the dilemma and which determines its possible varieties. For if only the antecedent or only the consequent be more than one, we must, in order to obtain a disjunctive minor, affirm the antecedent in the former case and deny the consequent in the latter; whereas, if there be more than one of both, it is open to us to take either course. This gives us four types of dilemma.

(1) *Simple Constructive.*

§ 383.   If A is B or C is D, E is F.

Either A is B or C is D.

∴ E is F.

If it is Wednesday or Friday, there is fish for dinner.

It is either Wednesday or Friday.

∴ There is fish for dinner.

(2) *Simple Destructive.*

If A is B, C is D and E is F.

Either C is not D or E is not F.

∴ A is not B.

If I go with Miss Brown to the ball, I must pay for her ticket and for my own.

Either I cannot pay for her ticket or I cannot pay for my own.

∴ I cannot go with Miss Brown to the ball.

(3) *Complex Constructive.*

If A is B, C is D; and if E is F, G is H.

Either A is B   or E is F.

∴ Either   C is D   or   G is H.

If I cross the field, I shall meet the bull; and, if I go up the lane, I shall meet the farmer.

Either I must cross the field or go up the lane.

∴ Either I shall meet the bull or the farmer.

(4) *Complex Destructive.*

If A is B, C is D; and if E is F, G is H.

Either   C is not D   or G is not H.

∴ Either A is not B   or E is not F.

If he were clever, he would see his mistake ; and if
he were candid, he would acknowledge it.

Either he does not see his mistake or he will not
acknowledge it.

∴ Either he is not clever or he is not candid.

§ 384. A Simple Dilemma is one of which the con-
clusion is a simple proposition.

A Complex Dilemma is one of which the conclusion is
a disjunctive proposition.

There is a slight inconvenience attending this ter-
minology, for even a simple dilemma is still a complex
syllogism.

§ 385. We will now give an exact definition of the
dilemma—

A dilemma is a complex syllogism having for its major
premiss a conjunctive proposition with more than one
antecedent or more than one consequent or both, of
which in the minor the antecedent is disjunctively asserted
or the consequent disjunctively denied.

§ 386. It must be noticed that the simple destructive
dilemma would not admit of a disjunctive consequent.
If we said

If A is B, C is D or E is F,
Either C is not D or E is not F,

we should not be denying the consequent. For ' E is
not F ' would make it true that C is D and ' C is not D '
would make it true that E is F ; so that in either case
we should have one of the alternatives true, which is just

what the disjunctive form 'Either C is D or E is F' insists upon. To apply this to the concrete instance, If I have only to pay for Miss Brown's ticket or for my own, I can still go with her to the ball [1].

§ 387. The several members of a complex dilemma, instead of being distributively assigned to one another, may be connected together as a whole, thus—

> If either A is B or E is F, either C is D or G is H.
> Either A is B or E is F.
> ∴ Either C is D or G is H.

In this shape the likeness of the dilemma to the semi-conjunctive syllogism becomes apparent. The major premiss is now vaguer than before. For each antecedent has a disjunctive choice of consequents, instead of being limited to one. This vagueness however does not affect the conclusion. For, so long as the conclusion is established, it does not matter from which members of the major its own members flow.

The complex destructive dilemma may be treated in the same way—

> If either A is B or E is F, either C is D or G is H.
> Either C is not D or G is not H.
> ∴ Either A is not B or E is not F.

---

[1] The authors of the *Palaestra Logica* challenge the right of the simple destructive dilemma to be called a dilemma at all, on the ground that the minor premiss would be better expressed thus— It cannot be *both* that C is D and that E is F. But, to be consistent, they should expunge also the complex destructive dilemma, to which the same remark applies.

The Polylemma.

§ 388. For the sake of simplicity we have limited the examples to the case of two antecedents or consequents. But we may have as many of either as we please, so as to have a Trilemma, a Tetralemma, and so on.

Trilemma.

If A is B, C is D ; and if E is F, G is H ; and if K is L, M is N.
Either A is B   or E is F   or   K is L.
∴ Either   C is D   or G is H   or   M is N.

Refutation of the Dilemma.

§ 389. So complex a piece of reasoning as the dilemma naturally affords many points of attack. We may deny the conjunction between antecedent and consequent assumed in the major, or the exhaustiveness of the alternatives implied in the minor, or we may detect some ambiguity in the terms, or we may find a flaw in the reasoning itself. Take for instance the dilemma by which the Stoics sought to fortify themselves against the idea of pain—

Levis est, si ferre possum ; brevis est, si ferre non possum.
(Seneca, *Epist.* xxiv, § 14) [1],

which may be formulated thus—

If I can bear pain, it is slight; if I cannot bear it, it is brief.

Either I can bear it or not.

∴ Pain is either slight or brief.

[1] Cp. Cic. *Fin.* I, § 40; II, §§ 22, 94, 95 : *T. D.* III, § 38; V, § 88.

Here the major denies what is, fortunately or unfortunately, a fact, that the human body is capable of enduring intense and prolonged pain.

The weak point in a dilemma is usually the minor premiss, where there is a tacit assumption that the alternatives offered are exclusive and exhaustive. Thus the dilemma against examinations—

If students are idle, examinations are unavailing; and if they are industrious, examinations are superfluous.

Students are either idle or industrious.

∴ Examinations are either unavailing or superfluous—

leaves out of sight the commonest case of all, that of students who will be industrious, if they have an immediate motive for exertion, but idle, if they have not.

The Rebutting of a Dilemma.

§ 390. A dilemma is said to be rebutted or retorted, when another dilemma is made out which apparently proves an opposite conclusion. This is a dialectical fallacy based on the fact, which is generally overlooked, that a disjunctive denial is not really opposed to a disjunctive assertion.

The most usual mode of rebutting a complex dilemma is by transposing and denying the consequents in the major.

If A is B, C is D; and if E is F, G is H.
Either A is B   or E is F.
∴ Either   C is D   or   G is H.

The same rebutted —

    If A is B, G is not H ; and if E is F, C is not D.
    Either A is B or E is F.

∴ Either G is not H or C is not D.

= Either C is not D or G is not H.

Under this form comes the dilemma addressed by the Athenian mother to her son—

'Do not enter upon public life : for, if you say what is just, men will hate you ; and if you say what is unjust, the gods will hate you '—

together with the retort made to it,

'I ought to enter upon public life : for, if I say what is just, the gods will love me ; and if I say what is unjust, men will love me.'

But the two conclusions here are quite compatible. A man must, on the given premises, be both hated and loved, whatever course he takes. So far indeed are two propositions of the form

        Either C is D or G is H,
    and Either C is not D or G is not H,

from being incompatible, that they express precisely the same thing when contradictory alternatives have been selected, e. g.

    Either a triangle is equilateral or non-equilateral.
    Either a triangle is non-equilateral or equilateral.

Equally illusory is the famous instance of rebutting

a dilemma contained in the story of Protagoras and Euathlus (Aul. Gell. *Noct. Att.* v. 10).

Euathlus was a pupil of Protagoras in rhetoric. He paid half the fee demanded by his preceptor before receiving lessons, and agreed to pay the remainder when he won his first case. But as he never proceeded to practise at the bar, it became evident that he meant to bilk his tutor. Accordingly Protagoras himself instituted a lawsuit against him, and in the preliminary proceedings before the jurors propounded to him the following dilemma—

'Most foolish young man, whatever be the issue of this suit, you must pay me what I claim : for, if the verdict be given in your favour, you are bound by our bargain ; and if it be given against you, you are bound by the decision of the jurors.'

But Protagoras had not taught his pupil to no purpose ; for he rebutted the dilemma as follows—

'Most sapient master, whatever be the issue of this suit, I shall not pay you what you claim : for, if the verdict be given in my favour, I am absolved by the decision of the jurors ; and if it be given against me, I am absolved by our bargain.'

The jurors are said to have been so puzzled by the conflicting plausibility of the arguments that they adjourned the case till the Greek Kalends. It is evident however that a grave injustice was thus done to Protagoras. His dilemma was really invincible. In the counter-dilemma of Euathlus we are meant to infer that Protagoras would

actually lose his fee, instead of merely getting it in one way rather than another. In either case he would both get and lose his fee, in the sense of getting it on one plea and not getting it on another: but in neither case would he actually lose it.

CHAPTER XXXIII.

Of Trains of Reasoning.

§ 391. The formal logician is only concerned to examine whether the conclusion duly follows from the premisses: he need not concern himself with the truth or falsity of his data. But the premisses of one syllogism may themselves be conclusions deduced from other syllogisms, the premisses of which may in their turn have been established by yet earlier syllogisms. When syllogisms are thus linked together we have what is called a Train of Reasoning.

§ 392. It is plain that all truths cannot be established by reasoning. For the attempt to do so would involve us in an infinite regress, wherein the number of syllogisms required would increase at each step in a geometrical ratio. To establish the premisses of a given syllogism we should require two preceding syllogisms; to establish their premisses, four; at the next step backwards, eight; at the next, sixteen; and so on *ad infinitum*. Thus the very possibility of reasoning implies truths that are known to us prior to all reasoning; and, however long a train of reasoning may be, we must ultimately come to truths which are either self-evident or are taken for granted.

§ 393. Any syllogism which establishes one of the premisses of another is called in reference to that other a Pro-syllogism, while a syllogism which has for one of its premisses the conclusion of another syllogism is called in reference to that other an Epi-syllogism.

The Epicheirema.

§ 394. The name Epicheirema is given to a syllogism with one or both of its premisses supported by a reason. Thus the following is a double epicheirema—

> All B is A, for it is E.
> All C is B, for it is F.
> ∴ All C is A.

> All virtue is praiseworthy, for it promotes the general welfare.
> Generosity is a virtue, for it prompts men to postpone self to others.
> ∴ Generosity is praiseworthy.

§ 395. An epicheirema is said to be of the first or second order according as the major or minor premiss is thus supported. The double epicheirema is a combination of the two orders [1].

§ 396. An epicheirema, it will be seen, consists of one syllogism fully expressed together with one, or, it may be two enthymemes (§ 269). In the above instance, if the reasoning which supports the premisses were set forth at

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[1] This, Cicero tells us, is the full type of rhetorical reasoning. *De Inv.* I, § 67.

full length, we should have, in place of the enthymemes, the two following pro-syllogisms —

> (1) All E is A.
> All B is E.
> ∴ All B is A.

Whatever promotes the general welfare is praise-
worthy.

Every virtue promotes the general welfare.

∴ Every virtue is praiseworthy.

> (2) All F is B.
> All C is F.
> ∴ All C is B.

Whatever prompts men to postpone self to others is
a virtue.

Generosity prompts men to postpone self to others.

∴ Generosity is a virtue.

§ 397. The enthymemes in the instance above given
are both of the first order, having the major premiss
suppressed. But there is nothing to prevent one or both
of them from being of the second order—

> All B is A, because all D is.
> All C is B, because all E is.
> ∴ All C is A.

All Mahometans are fanatics, because all Monotheists
are.

These men are Mahometans, because all Persians
are.

∴ These men are fanatics.

Here it is the minor premiss in each syllogism that is suppressed, namely,

(1) All Mahometans are Monotheists.

(2) These men are Persians.

The Sorites.

§ 398. The Sorites is the neatest and most compendious form that can be assumed by a train of reasoning.

It is sometimes more appropriately called the chain-argument[1], and may be defined as—

A train of reasoning, in which one premiss of each epi-syllogism is supported by a pro-syllogism, the other being taken for granted.

This is its inner essence.

§ 399. In its outward form it may be described as—A series of propositions, each of which has one term in common with that which preceded it, while in the conclusion one of the terms in the last proposition becomes either subject or predicate to one of the terms in the first.

§ 400. A sorites may be either—

(1) Progressive,

or (2) Regressive.

Progressive Sorites.	*Regressive Sorites.*
All A is B.	All D is E.
All B is C.	All C is D.

[1] The transference of the name Sorites to the chain-argument is perhaps due to Cic. *Fin.* IV, § 50.

All C is D.	All B is C.
All D is E.	All A is B.
∴ All A is E.	∴ All A is E.

The usual form is the progressive; so that the sorites is commonly described as a series of propositions in which the predicate of each becomes the subject of the next, while in the conclusion the last predicate is affirmed or denied of the first subject. The regressive form however exactly reverses these attributes; and would require to be described as a series of propositions, in which the subject of each becomes the predicate of the next, while in the conclusion the first predicate is affirmed or denied of the last subject.

The regressive sorites, it will be observed, consists of the same propositions as the progressive one, only written in reverse order. Why then, it may be asked, do we give a special name to it, though we do not consider a syllogism different, if the minor premiss happens to precede the major? It is because the sorites is not a mere series of propositions, but a compressed train of reasoning; and the two trains of reasoning may be resolved into their component syllogisms in such a manner as to exhibit a real difference between them.

The Progressive Sorites is a train of reasoning in which the minor premiss of each epi-syllogism is supported by a pro-syllogism, while the major is taken for granted.

The Regressive Sorites is a train of reasoning in which the major premiss of each epi-syllogism is supported by a pro-syllogism, while the minor is taken for granted.

Progressive Sorites.	*Regressive Sorites.*
(1) All B is C.	(1) All D is E.
All A is B.	All C is D.
∴ All A is C.	∴ All C is E.
(2) All C is D.	(2) All C is E.
All A is C.	All B is C.
∴ All A is D.	∴ All B is E.
(3) All D is E.	(3) All B is E.
All A is D.	All A is B.
∴ All A is E.	∴ All A is E.

§ 401. Here is a concrete example of the two kinds of sorites, resolved each into its component syllogisms—

Progressive Sorites.

All Bideford men are Devonshire men.
All Devonshire men are Englishmen.
All Englishmen are Teutons.
All Teutons are Aryans.
∴ All Bideford men are Aryans.

(1) All Devonshire men are Englishmen.
All Bideford men are Devonshire men.
∴ All Bideford men are Englishmen.

(2) All Englishmen are Teutons.
All Bideford men are Englishmen.
∴ All Bideford men are Teutons.

(3) All Teutons are Aryans.
All Bideford men are Teutons.
∴ All Bideford men are Aryans.

Regressive Sorites.

All Teutons are Aryans.
All Englishmen are Teutons.
All Devonshiremen are Englishmen.
All Bideford men are Devonshiremen.
∴ All Bideford men are Aryans.

(1) All Teutons are Aryans.
All Englishmen are Teutons.
∴ All Englishmen are Aryans.

(2) All Englishmen are Aryans.
All Devonshiremen are Englishmen.
∴ All Devonshiremen are Aryans.

(3) All Devonshiremen are Aryans.
All Bideford men are Devonshiremen.
∴ All Bideford men are Aryans.

§ 402. When expanded, the sorites is found to contain as many syllogisms as there are propositions intermediate between the first and the last. This is evident also on inspection by counting the number of middle terms.

In expanding the progressive form we have to commence with the second proposition of the sorites as the major premiss of the first syllogism. In the progressive form the subject of the conclusion is the same in all the syllogisms ; in the regressive form the predicate is the same. In both the same series of means, or middle terms,

is employed, the difference lying in the extremes that are compared with one another through them.
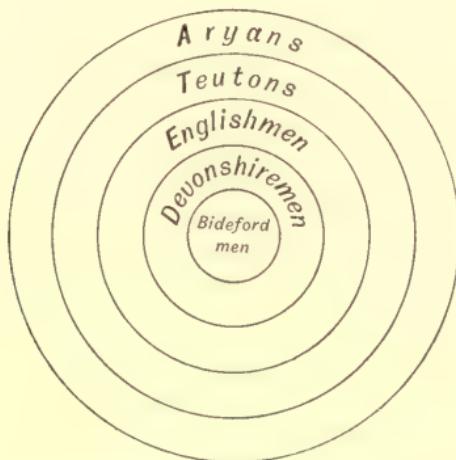
It is apparent from the figure that in the progressive form we work from within outwards, in the regressive form from without inwards.   In the former we first employ the term 'Devonshiremen' as a mean to connect 'Bideford men' with 'Englishmen;' next we employ 'Englishmen' as a mean to connect the same subject 'Bideford men' with the wider term 'Teutons;' and, lastly, we employ 'Teutons' as a mean to connect the original subject 'Bideford men' with the ultimate predicate 'Aryans.'

Reversely, in the regressive form we first use 'Teutons' as a mean whereby to bring 'Englishmen' under 'Aryans;' next we use 'Englishmen' as a mean whereby to bring 'Devonshiremen' under the same predicate 'Aryans;' and, lastly, we use 'Devonshiremen' as a mean whereby

to bring the ultimate subject 'Bideford men' under the original predicate 'Aryans.'

§ 403. A sorites may be either Regular or Irregular.

In the regular form the terms which connect each proposition in the series with its predecessor, that is to say, the middle terms, maintain a fixed relative position ; so that, if the middle term be subject in one, it will always be predicate in the other, and vice versâ. In the irregular form this symmetrical arrangement is violated.

The syllogisms which compose a regular sorites, whether progressive or regressive, will always be in the first figure.

In the irregular sorites the syllogisms may fall into different figures.

§ 404. For the regular sorites the following rules may be laid down.

> (1) Only one premiss can be particular, namely, the first, if the sorites be progressive, the last, if it be regressive.
>
> (2) Only cne premiss can be negative, namely, the last, if the sorites be progressive, the first, if it be regressive.

§ 405. *Proof of the Rules for the Regular Sorites.*

> (1) In the progressive sorites the proposition which stands first is the only one which appears as a minor premiss in the expanded form. Each of the others is used in its turn as a major. If any proposition, therefore, but the first were

particular, there would be a particular major, which involves undistributed middle, if the minor be affirmative, as it must be in the first figure.

In the regressive sorites, if any proposition except the last were particular, we should have a particular conclusion in the syllogism in which it occurred as a premiss, and so a particular major in the next syllogism, which again is inadmissible, as involving undistributed middle.

(2) In the progressive sorites, if any premiss before the last were negative, we should have a negative conclusion in the syllogism in which it occurs. This would necessitate a negative minor in the next syllogism, which is inadmissible in the first figure, as involving illicit process of the major.

In the regressive sorites the proposition which stands first is the only one which appears as a major premiss in the expanded form. Each of the others is used in its turn as a minor. If any premiss, therefore, but the first were negative, we should have a negative minor in the first figure, which involves illicit process of the major.

§ 406. The rules above given do not apply to the irregular sorites, except so far as that only one premiss can be particular and only one negative, which follows

from the general rules of syllogism. But there is nothing
to prevent any one premiss from being particular or any
one premiss from being negative, as the subjoined ex-
amples will show. Both the instances chosen belong to
the progressive order of sorites.

<div style="text-align:center">

(1)   **Barbara.**

</div>

All B is A.	All B is A.
All C is B.	All C is B.
Some C is D.	∴ All C is A.
All D is E.	
∴ Some A is E.	

<div style="text-align:center">

(2)   **Disamis.**

Some C is D.
All C is A.
∴ Some A is D.

</div>

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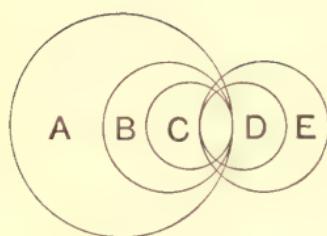
(3)   **Darii.**

All D is E.
Some A is D.
∴ Some A is E.

</div>

All A is B.	(1) **Barbara.**
All B is C.	All B is C.
No D is C.	All A is B.
All E is D.	∴ All A is C.
∴ No A is E.	
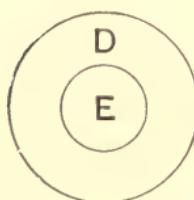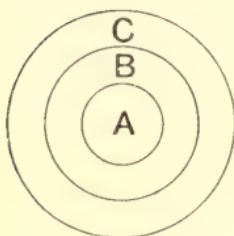
(2)   **Cesare.**
No D is C.
All A is C.
∴ No A is D.

(3)   **Camestres.**
All E is D.
No A is D.
∴ No A is E.

§ 407. A chain-argument may be composed consisting of conjunctive instead of simple propositions. This is subject to the same laws as the simple sorites, to which it is immediately reducible.

Progressive.	*Regressive.*
If A is B, C is D.	If E is F, G is H.
If C is D, E is F.	If C is D, E is F.
If E is F, G is H.	If A is B, C is D.
∴ If A is B, G is H.	∴ If A is B, G is H.

CHAPTER XXXIV.

Of Fallacies.

§ 408. AFTER examining the conditions on which correct
thoughts depend, it is expedient to classify some of the
most familiar forms of error. It is by the treatment of
the Fallacies that logic chiefly vindicates its claim to be
considered a practical rather than a speculative science.
To explain and give a name to fallacies is like setting
up so many sign-posts on the various turns which it is
possible to take off the road of truth.

§ 409. By a fallacy is meant a piece of reasoning which
appears to establish a conclusion without really doing so.
The term applies both to the legitimate deduction of a
conclusion from false premisses and to the illegitimate
deduction of a conclusion from any premisses. There
are errors incidental to conception and judgement, which
might well be brought under the name; but the fallacies
with which we shall concern ourselves are confined to
errors connected with inference.

§ 410. When any inference leads to a false conclusion, the error may have arisen either in the thought itself or in the signs by which the thought is conveyed. The main sources of fallacy then are confined to two—

> (1) thought,
> (2) language.

§ 411. This is the basis of Aristotle's division of fallacies, which has not yet been superseded. Fallacies, according to him, are either in the language or outside of it. Outside of language there is no source of error but thought. For things themselves do not deceive us, but error arises owing to a misinterpretation of things by the mind. Thought however may err either in its form or in its matter. The former is the case where there is some violation of the laws of thought; the latter whenever thought disagrees with its object. Hence we arrive at the important distinction between Formal and Material fallacies, both of which, however, fall under the same negative head of fallacies other than those of language.

$$
\text{Fallacy}
\begin{cases}
\text{In the language} \\
\text{(in the signs of thought).} \\
\\
\text{Outside the language} \\
\text{(in the thought itself)}
\begin{cases}
\text{In the Form.} \\
\\
\text{In the Matter.}
\end{cases}
\end{cases}
$$
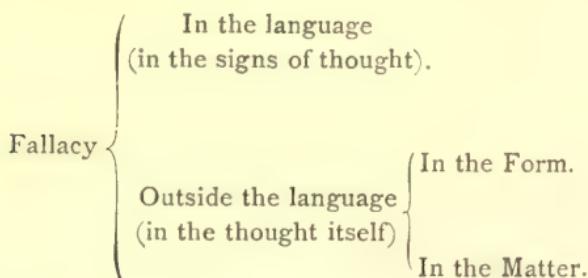
§ 412. There are then three heads to which fallacies may be referred—namely, Formal Fallacies, Fallacies of

Language, which are commonly known as Fallacies of Ambiguity, and, lastly, Material Fallacies.

§ 413. Aristotle himself only goes so far as the first step in the division of fallacies, being content to class them according as they are in the language or outside of it. After that he proceeds at once to enumerate the *infimae species* under each of the two main heads. We shall presently imitate this procedure for reasons of expediency. For the whole phraseology of the subject is derived from Aristotle's treatise on Sophistical Refutations, and we must either keep to his method or break away from tradition altogether. Sufficient confusion has already arisen from retaining Aristotle's language while neglecting his meaning.

§ 414. Modern writers on logic do not approach fallacies from the same point of view as Aristotle. Their object is to discover the most fertile sources of error in solitary reasoning; his was to enumerate the various tricks of refutation which could be employed by a sophist in controversy. Aristotle's classification is an appendix to the Art of Dialectic.

§ 415. Another cause of confusion in this part of logic is the identification of Aristotle's twofold division of fallacies, commonly known under the titles of *In dictione* and *Extra dictionem*, with the division into Logical and Material, which is based on quite a different principle.

Aristotle's division perhaps allows an undue importance to language, in making that the principle of division,

and so throwing formal and material fallacies under a common head. Accordingly another classification has been adopted, which concentrates attention from the first upon the process of thought, which ought certainly to be of primary importance in the eyes of the logician. This classification is as follows.

Whenever in the course of our reasoning we are involved in error, either the conclusion follows from the premisses or it does not. If it does not, the fault must lie in the process of reasoning, and we have then what is called a Logical Fallacy. If, on the other hand, the conclusion does follow from the premisses, the fault must lie in the premisses themselves, and we then have what is called a Material Fallacy. Sometimes, however, the conclusion will appear to follow from the premisses until the meaning of the terms is examined, when it will be found that the appearance is deceptive owing to some ambiguity in the language. Such fallacies as these are, strictly speaking, non-logical, since the meaning of words is extraneous to the science which deals with thought. But they are called Semi-logical. Thus we arrive by a different road at the same three heads as before, namely, (1) Formal or Purely Logical Fallacies, (2) Semi-logical Fallacies or Fallacies of Ambiguity, (3) Material Fallacies.

§ 416. For the sake of distinctness we will place the two divisions side by side, before we proceed to enumerate the *infimae species*.

$$
\text{Fallacy}
\begin{cases}
\text{In the language} \\
\text{(Fallacy of Ambiguity).} \\
\\
\text{Outside the language}
\begin{cases}
\text{In the Form.} \\
\text{In the Matter.}
\end{cases}
\end{cases}
$$

$$
\text{Fallacy}
\begin{cases}
\text{Logical}
\begin{cases}
\text{Formal} \\
\text{or purely logical.} \\
\\
\text{Semi-logical} \\
\text{(Fallacy of Ambiguity).}
\end{cases} \\
\\
\text{Material.}
\end{cases}
$$
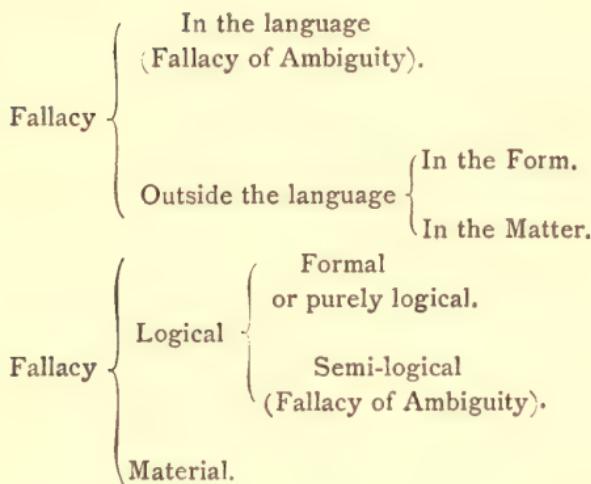
§ 417. Of one of these three heads, namely, formal fallacies, it is not necessary to say much, as they have been amply treated of in the preceding pages. A formal fallacy arises from the breach of any of the general rules of syllogism. Consequently it would be a formal fallacy to present as a syllogism anything which had more or less than two premisses. Under the latter variety comes what is called 'a woman's reason,' which asserts upon its own evidence something which requires to be proved. When the conclusion thus merely reasserts one of the premisses, the other must be either absent or irrelevant. If, on the other hand, there are more than two premisses, either there is more than one syllogism or the superfluous premiss is no premiss at all, but a proposition irrelevant to the conclusion.

§ 418. The remaining rules of the syllogism are more liable to be broken than the first; so that the following

scheme presents the varieties of formal fallacy which are commonly enumerated—

Formal Fallacy $\begin{cases} \text{Four Terms.} \\ \text{Undistributed Middle.} \\ \text{Illicit Process.} \\ \text{Negative Premisses and Conclusion.} \end{cases}$

§ 419. The Fallacy of Four Terms is a violation of the second of the general rules of syllogism (§ 285). Here is a palpable instance of it—

> All men who write books are authors.
> All educated men could write books.
> ∴ All educated men are authors.

Here the middle term is altered in the minor premiss to the destruction of the argument. The difference between the actual writing of books and the power to write them is precisely the difference between one who is an author and one who is not.

The Fallacies of Undistributed Middle and Illicit Process have been treated of under § 287. The heading ' Negative Premisses and Conclusion ' covers violations of the general rules of syllogism relating to negative premisses (§ 285). Here is an instance of the particular form of the fallacy which consists in the attempt to extract an affirmative conclusion out of two negative premisses—

> All salmon are fish, for neither salmon nor fish belong to the class mammalia.

The accident of a conclusion being true often helps to conceal the fact that it is illegitimately arrived at.

The formal fallacies which have just been enumerated find no place in Aristotle's division. The reason is plain. His object was to enumerate the various modes in which a sophist might snatch an apparent victory, whereas by openly violating any of the laws of syllogism a disputant would be simply courting defeat.

§ 420. We now revert to Aristotle's classification of fallacies, or rather of Modes of Refutation. We will take the species he enumerates in their order, and notice how modern usage has departed from the original meaning of the terms. Let it be borne in mind that, when the deception was not in the language, Aristotle did not trouble himself to determine whether it lay in the matter or in the form of thought.

The following scheme presents the Aristotelian classification to the eye at a glance :—

Modes of Refutation

¹ In the language
- Equivocation.
- Amphiboly.
- Composition.
- Division.
- Accent.
- Figure of Speech.

Outside the language
- Accident.
- *A dicto secundum quid.*
- *Ignoratio Elenchi.*
- Consequent.
- *Petitio Principii.*
- *Non causa pro causa.*
- Many Questions.

¹ The Greek is παρὰ τὴν λέξιν, the exact meaning of which is 'due to the statement.' The Stoics spoke of τῶν παρὰ τὴν φωνὴν καὶ τὰ πράγματα σοφισμάτων. D. L. VII, § 43.

§ 421. The Fallacy of Equivocation (ὁμωνυμία) consists in an ambiguous use of any of the three terms of a syllogism. If, for instance, anyone were to argue thus—

> No human being is made of paper,
>
> All pages are human beings,
>
> ∴ No pages are made of paper—

the conclusion would appear paradoxical, if the minor term were there taken in a different sense from that which it bore in its proper premiss. This therefore would be an instance of the fallacy of Equivocal Minor.

For a glaring instance of the fallacy of Equivocal Major, we may take the following—

> No courageous creature flies,
>
> The eagle is a courageous creature,
>
> ∴ The eagle does not fly—

the conclusion here becomes unsound only by the major being taken ambiguously.

It is however to the middle term that an ambiguity most frequently attaches. In this case the fallacy of equivocation assumes the special name of the Fallacy of Ambiguous Middle. Take as an instance the following—

> Faith is a moral virtue.
>
> To believe in the Book of Mormon is faith.
>
> ∴ To believe in the Book of Mormon is a moral virtue.

Here the premisses singly might be granted; but the conclusion would probably be felt to be unsatisfactory. Nor is the reason far to seek. It is evident that belief in

a book cannot be faith in any sense in which that quality can rightly be pronounced to be a moral virtue.

§ 422. The Fallacy of Amphiboly (ἀμφιβολία) is an ambiguity attaching to the construction of a proposition rather than to the terms of which it is composed[1]. One of Aristotle's examples is this—

τὸ βούλεσθαι λαβεῖν με τοὺς πολεμίους,

which may be interpreted to mean either ' the fact of my wishing to take the enemy,' or 'the fact of the enemies' wishing to take me.' The classical languages are especially liable to this fallacy owing to the oblique construction in which the accusative becomes subject to the verb. Thus in Latin we have the oracle given to Pyrrhus (though of course, if delivered at all, it must have been in Greek)—

> Aio te, Aeacida, Romanos vincere posse[2].
> Pyrrhus the Romans shall, I say, subdue (*Whately*),

which Pyrrhus, as the story runs, interpreted to mean that he could conquer the Romans, whereas the oracle subsequently explained to him that the real meaning was that the Romans could conquer him. Similar to this, as Shakspeare makes the Duke of York point out, is the witch's prophecy in *Henry VI* (Second Part, Act i, sc. 4),

> The duke yet lives that Henry shall depose.

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[1] A Stoic example of ἀμφιβολία is αὐλὴ τρὶς πέπτωκε (= The house has fallen three times, or, The flute-girl has had a fall), which would rather come under the head of Aristotle's ' division,' § 423. See D. L. VII, § 62.

[2] Cicero, *De Div.* ii, § 116 ; Quintilian, *Inst. Orat.* vii. 9, § 6.

An instance of amphiboly may be read on the walls of Windsor Castle—*Hoc fecit Wykeham.* The king was incensed with the bishop for daring to record that he made the tower, but the latter adroitly replied that what he really meant to indicate was that the tower was the making of him. To the same head may be referred the famous sentence—'I will wear no clothes to distinguish me from my Christian brethren.'

§ 423. The Fallacy of Composition (σύνθεσις) is likewise a case of ambiguous construction. It consists, as expounded by Aristotle, in taking words together which ought to be taken separately, e. g.

'Is it possible for a man who is not writing to write?' 'Of course it is.' 'Then it is possible for a man to write without writing.'

And again—

'Can you carry this, that, and the other?' 'Yes.'

'Then you can carry this, that, and the other,'— a fallacy against which horses would protest, if they could.

It is doubtless this last example which has led to a convenient misuse of the term 'fallacy of composition' among modern writers, by whom it is defined to consist in arguing from the distributive to the collective use of a term.

The Fallacy of Division (διαίρεσις), on the other hand, consists in taking words separately which ought to be taken together, e. g.

ἐγώ σ' ἔθηκα δοῦλον ὄντ' ἐλεύθερον[1],

---

[1] Evidently the original of the line in Terence's *Andria*, 37—
feci ex servo ut esses libertus mihi.

where the separation of δοῦλον from ὄντα would lead to an interpretation exactly contrary to what is intended.

And again—

πεντήκοντ᾽ ἀνδρῶν ἑκατὸν λίπε δῖος Ἀχιλλεύς,

where the separation of ἀνδρῶν from ἑκατόν leads to a ludicrous error.

Any reader whose youth may have been nourished on 'The Fairchild Family' may possibly recollect a sentence which ran somewhat on this wise—'Henry,' said Mr. Fairchild, 'is this true? Are you a thief and a liar too?' But I am afraid he will miss the keen delight which can be extracted at a certain age from turning the tables upon Mr. Fairchild thus—Henry said, 'Mr. Fairchild, is this true? Are *you* a thief and a liar too?'

§ 424. The fallacy of division has been accommodated by modern writers to the meaning which they have assigned to the fallacy of composition. So that by the 'fallacy of division' is now meant arguing from the collective to the distributive use of a term. Further, it is laid down that when the middle term is used distributively in the major premiss and collectively in the minor, we have the fallacy of composition; whereas, when the middle term is used collectively in the major premiss and distributively in the minor, we have the fallacy of division. Thus the first of the two examples appended would be composition and the second division.

(1) Two and three are odd and even.
   Five is two and three.
   ∴ Five is odd and even.

(2) The Germans are an intellectual people.
   Hans and Fritz are Germans.
∴ They are intellectual people.

As the possibility of this sort of ambiguity is not
confined to the middle term, it seems desirable to add that
when either the major or minor term is used distributively
in the premiss and collectively in the conclusion, we have
the fallacy of composition, and in the converse case the
fallacy of division.   Here is an instance of the latter kind
in which the minor term is at fault—

Anything over a hundredweight is too heavy to lift.
These sacks (collectively) are over a hundred-
   weight.
∴ These sacks (distributively) are too heavy to lift.

The ambiguity of the word 'all' is a great assistance
in the English language to the pair of fallacies just
spoken of.

§ 425. The Fallacy of Accent ($\pi\rho\sigma\sigma\omega\delta\iota\alpha$) is neither more
nor less than a mistake in Greek accentuation.   As an
instance Aristotle gives *Iliad* xxiii. 328, where the ancient
copies of Homer made nonsense of the words τὸ μὲν οὐ
καταπύθεται ὄμβρῳ by writing οὗ with the circumflex in
place of οὔ with the acute accent[1].   Aristotle remarks
that the fallacy is one which cannot easily occur in verbal
argument, but rather in writing and poetry.

---

[1] This goes to show that the ancient Greeks did not distinguish
in pronunciation between the rough and smooth breathing any
more than their modern representatives.

Modern writers explain the fallacy of accent to be the mistake of laying the stress upon the wrong part of a sentence. Thus when the country parson reads out, 'Thou shalt not bear false witness *against* thy neighbour,' with a strong emphasis upon the word 'against,' his ignorant audience leap to the conclusion that it is not amiss to tell lies, provided they be in favour of one's neighbour.

§ 426. The Fallacy of Figure of Speech (τὸ σχῆμα τῆς λέξεως) results from any confusion of grammatical forms, as between the different genders of nouns or the different voices of verbs, or their use as transitive or intransitive, e. g. ὑγιαίνειν has the same grammatical form as τέμνειν or οἰκοδομεῖν, but the former is intransitive, while the latter are transitive. A sophism of this kind is put into the mouth of Socrates by Aristophanes in the Clouds (670–80). The philosopher is there represented as arguing that κάρδοπος must be masculine because Κλεώνυμος is. On the surface this is connected with language, but it is essentially a fallacy of false analogy.

To this head may be referred what is known as the Fallacy of Paronymous Terms. This is a species of equivocation which consists in slipping from the use of one part of speech to that of another, which is derived from the same source, but has a different meaning. Thus this fallacy would be committed if, starting from the fact that there is a certain probability that a hand at whist will consist of thirteen trumps, one were to proceed to argue that it was probable, or that he had proved it.

§ 427. We turn now to the tricks of refutation which lie outside the language, whether the deception be due to the assumption of a false premiss or to some unsoundness in the reasoning.

§ 428. The first on the list is the Fallacy of Accident (τὸ συμβεβηκός). This fallacy consists in confounding an essential with an accidental difference, which is not allowable, since many things are the same in essence, while they differ in accidents. Here is the sort of example that Aristotle gives—

'Is Plato different from Socrates?' 'Yes.' 'Is Socrates a man?' 'Yes.' 'Then Plato is different from man.'

To this we answer—No: the difference of accidents between Plato and Socrates does not go so deep as to affect the underlying essence. To put the thing more plainly, the fallacy lies in assuming that whatever is different from a given subject must be different from it in all respects, so that it is impossible for them to have a common predicate. Here Socrates and Plato, though different from one another, are not so different but that they have the common predicate 'man.' The attempt to prove that they have not involves an illicit process of the major.

§ 429. The next fallacy suffers from the want of a convenient name. It is called by Aristotle τὸ ἁπλῶς τόδε ἢ πῃ λέγεσθαι καὶ μὴ κυρίως or, more briefly, τὸ ἁπλῶς ἢ μή, or τὸ πῇ καὶ ἁπλῶς, and by the Latin writers 'Fallacia a dicto secundum quid ad dictum simpliciter.' It consists in taking what is said in a particular respect as though it held

true without any restriction, e. g. that because the non-existent (τὸ μὴ ὄν) is a matter of opinion, that therefore the non-existent is, or again that because the existent (τὸ ὄν) is not a man, that therefore the existent is not. Or again, if an Indian, who as a whole is black, has white teeth, we should be committing this species of fallacy in declaring him to be both white and not-white. For he is only white in a certain respect (πῇ), but not absolutely (ἁπλῶς). More difficulty, says Aristotle, may arise when opposite qualities exist in a thing in about an equal degree. When, for instance, a thing is half white and half black, are we to say that it is white or black? This question the philosopher propounds, but does not answer. The force of it lies in the implied attack on the Law of Contradiction. It would seem in such a case that a thing may be both white and not-white at the same time. The fact is—so subtle are the ambiguities of language—that even such a question as 'Is a thing white or not-white?' straightforward as it seems, is not really a fair one. We are entitled sometimes to take the bull by the horns, and answer with the adventurous interlocutor in one of Plato's dialogues— 'Both and neither.' It may be both in a certain respect, and yet neither absolutely.

The same sort of difficulties attach to the Law of Excluded Middle, and may be met in the same way. It might, for instance, be urged that it could not be said with truth of the statue seen by Nebuchadnezzar in his dream either that it was made of gold or that it was not made of gold: but the apparent plausibility of the objec-

tion would be due merely to the ambiguity of language. It is not true, on the one hand, that it was made of gold (in the sense of being composed entirely of that metal); and it is not true, on the other, that it was not made of gold (in the sense of no gold at all entering into its composition). But let the ambiguous proposition be split up into its two meanings, and the stringency of the Law of Excluded Middle will at once appear—

(1) It must either have been composed entirely of gold or not.

(2) Either gold must have entered into its composition or not.

By some writers this fallacy is treated as the converse of the last, the fallacy of accident being assimilated to it under the title of the 'Fallacia a dicto simpliciter ad dictum secundum quid.' In this sense the two fallacies may be defined thus.

The Fallacy of Accident consists in assuming that what holds true as a general rule will hold true under some special circumstances which may entirely alter the case.

The Converse Fallacy of Accident consists in assuming that what holds true under some special circumstances must hold true as a general rule.

The man who, acting on the assumption that alcohol is a poison, refuses to take it when he is ordered to do so by the doctor, is guilty of the fallacy of accident; the man who, having had it prescribed for him when he was ill,

continues to take it morning, noon, and night, commits the converse fallacy.

There ought to be added a third head to cover the fallacy of arguing from one special case to another.

§ 430. The next fallacy is *Ignoratio Elenchi* (ἐλέγχου ἄγνοια). This fallacy arises when by reasoning valid in itself one establishes a conclusion other than what is required to upset the adversary's assertion. It is due to an inadequate conception of the true nature of refutation. Aristotle therefore is at the pains to define refutation at full length, thus—

'A refutation (ἔλεγχος) is the denial of one and the same—not name, but thing, and by means, not of a synonymous term, but of the same term, as a necessary consequence from the data, without assumption of the point originally at issue, in the same respect, and in the same relation, and in the same way, and at the same time.'

The *elenchus* then is the exact contradictory of the opponent's assertion under the terms of the law of contradiction. To establish by a syllogism, or series of syllogisms, any other proposition, however slightly different, is to commit this fallacy. Even if the substance of the contradiction be established, it is not enough unless the identical words of the opponent are employed in the contradictory. Thus, if his thesis asserts or denies something about λώπιον, it is not enough for you to prove the contradictory with regard to ἱμάτιον. There will be need of a further question and answer to identify the two,

though they are admittedly synonymous. Such was the rigour with which the rules of the game of dialectic were enforced among the Greeks!

Under the head of ‘Ignoratio Elenchi’ it has become usual to speak of various forms of argument which have been labelled by the Latin writers under such names as ‘argumentum ad hominem,’ ‘ad populum,’ ‘ad verecundiam,’ ‘ad ignorantiam,’ ‘ad baculum’—all of them opposed to the ‘argumentum ad rem’ or ‘ad judicium.’

By the ‘argumentum ad hominem’ was perhaps meant a piece of reasoning which availed to silence a particular person, without touching the truth of the question. Thus a quotation from Scripture is sufficient to stop the mouth of a believer in the inspiration of the Bible. Hume’s Essay on Miracles is a noteworthy instance of the ‘argumentum ad hominem’ in this sense of the term. He insists strongly on the evidence for certain miracles which he knew that the prejudices of his hearers would prevent their ever accepting, and then asks triumphantly if these miracles, which are declared to have taken place in an enlightened age in the full glare of publicity, are palpably imposture, what credence can be attached to accounts of extraordinary occurrences of remote antiquity, and connected with an obscure corner of the globe? The ‘argumentum ad judicium’ would take miracles as a whole, and endeavour to sift the amount of truth which may lie in the accounts we have of them in every age [1].

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[1] On this subject see the author’s *Attempts at Truth* (B. H. Blackwell, Oxford), pp. 46-59.

In ordinary discourse at the present day the term 'argumentum ad hominem' is used for the form of irrelevancy which consists in attacking the character of the opponent instead of combating his arguments, as illustrated in the well-known instructions to a barrister—'No case: abuse the plaintiff's attorney.'

The 'argumentum ad populum' consists in an appeal to the passions of one's audience. An appeal to passion, or to give it a less question-begging name, to feeling, is not necessarily amiss. The heart of man is the instrument upon which the rhetorician plays, and he has to answer for the harmony or the discord that comes of his performance.

The 'argumentum ad verecundiam' is an appeal to the feeling of reverence or shame. It is an argument much used by the old to the young and by Conservatives to Radicals.

The 'argumentum ad ignorantiam' consists simply in trading on the ignorance of the person addressed, so that it covers any kind of fallacy that is likely to prove effective with the hearer.

The 'argumentum ad baculum' is unquestionably a form of irrelevancy. To knock a man down when he differs from you in opinion may prove your strength, but hardly your logic.

A sub-variety of this form of irrelevancy was exhibited lately at a socialist lecture in Oxford, at which an undergraduate, unable or unwilling to meet the arguments of the speaker, uncorked a bottle, which had the effect of instan-

taneously dispersing the audience. This might be set down as the 'argumentum ad nasum.'

§ 431. We now come to the Fallacy of the Consequent, a term which has been more hopelessly abused than any. What Aristotle meant by it was simply the assertion of the consequent in a conjunctive proposition, which amounts to the same thing as the simple conversion of A (§ 219), and is a fallacy of distribution. Aristotle's example is this—

> If it has rained, the ground is wet.
> ∴ If the ground is wet, it has rained.

This fallacy, he tells us, is often employed in rhetoric in dealing with presumptive evidence. Thus a speaker, wanting to prove that a man is an adulterer, will argue that he is a showy dresser, and has been seen about at nights. Both these things however may be the case, and yet the charge not be true.

§ 432. The Fallacy of 'Petitio' or 'Assumptio Principii' (τὸ ἐν ἀρχῇ αἰτεῖσθαι or λαμβάνειν) to which we now come, consists in an unfair assumption of the point at issue. The word αἰτεῖσθαι in Aristotle's name for it points to the Greek method of dialectic by means of question and answer. This fact is rather disguised by the mysterious phrase 'begging the question.' The fallacy would be committed when you asked your opponent to grant, overtly or covertly, the very proposition originally propounded for discussion.

As the question of the precise nature of this fallacy

is of some importance we will take the words of Aristotle himself (*Top.* viii. 13, §§ 2, 3): 'People seem to beg the question in five ways. First and most glaringly, when one takes for granted the very thing that has to be proved. This by itself does not readily escape detection, but in the case of "synonyms," that is, where the name and the definition have the same meaning, it does so more easily [1]. Secondly, when one assumes universally that which has to be proved in particular, as, if a man undertaking to prove that there is one science of contraries, were to assume that there is one science of opposites generally. For he seems to be taking for granted along with several other things what he ought to have proved by itself. Thirdly, when one assumes the particulars where the universal has to be proved; for in so doing a man is taking for granted separately what he was bound to prove along with several other things. Again, when one assumes the question at issue by splitting it up, for instance, if, when the point to be proved is that the art of medicine deals with health and disease, one were to take each by itself for granted. Lastly, if one were to take for granted one of a pair of necessary consequences, as that

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[1] Some light is thrown upon this obscure passage by a comparison with *Cat.* I, § 3, where 'synonym' is defined. To take the word here in its later and modern sense affords an easy interpretation, which is countenanced by Alexander Aphrodisiensis, but it is flat against the usage of Aristotle, who elsewhere gives the name 'synonym,' not to two names for the same thing, but to two things going under the same name. See Trendelenburg on the passage.

the side is incommensurable with the diagonal, when it is
required to prove that the diagonal is incommensurable
with the side.'

To sum up briefly, we may beg the question in five
ways—

(1) By simply asking the opponent to grant the point
which requires to be proved ;

(2) by asking him to grant some more general truth
which involves it ;

(3) by asking him to grant the particular truths which
it involves ;

(4) by asking him to grant the component parts of it
in detail ;

(5) by asking him to grant a necessary consequence
of it.

The first of these five ways, namely, that of begging
the question straight off, lands us in the formal fallacy
already spoken of (§ 417), which violates the first of the
general rules of syllogism, inasmuch as a conclusion is
derived from a single premiss, to wit, itself.

The second, strange to say, gives us a sound syllogism
in **Barbara**, a fact which countenances the blasphemers
of the syllogism in the charge they bring against it of
containing in itself a *petitio principii.* Certainly Aristotle's
expression might have been more guarded. But it is
clear that his quarrel is with the matter, not with the form
in such an argument. The fallacy consists in assuming
a proposition which the opponent would be entitled to

deny. Elsewhere Aristotle tells us that the fallacy arises when a truth not evident by its own light is taken to be so [1].

The third gives us an *inductio per enumerationem simplicem*, a mode of argument which would of course be unfair as against an opponent who was denying the universal.

The fourth is a more prolix form of the first.

The fifth rests on Immediate Inference by Relation (§ 238).

Under the head of *petitio principii* comes the fallacy of Arguing in a Circle, which is incidental to a train of reasoning. In its most compressed form it may be represented thus—

(1)	B is A.	(2)	C is A.
	C is B.		B is C.
	∴ C is A.		∴ B is A.

§ 433. The Fallacy of *Non causa pro causa* (τὸ μὴ αἴτιον ὡς αἴτιον) is another, the name of which has led to a complete misinterpretation. It consists in importing a contradiction into the discussion, and then fathering it on the position controverted. Such arguments, says Aristotle, often impose upon the users of them themselves. The instance he gives is too recondite to be of general interest.

§ 434. Lastly, the Fallacy of Many Questions (τὸ τὰ

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[1] Ὅταν τὸ μὴ δι᾽ αὐτοῦ γνωστὸν δι᾽ αὐτοῦ τις ἐπιχειρῇ δεικνύναι, τότ᾽ αἰτεῖται τὸ ἐξ ἀρχῆς. *Anal. Pr.* II. 16, § 1 ad fin.

δύο ἐρωτήματα ἓν ποιεῖν) is a deceptive form of interrogation, when a single answer is demanded to what is not really a single question. In dialectical discussions the respondent was limited to a simple 'yes' or 'no;' and in this fallacy the question is so framed as that either answer would seem to imply the acceptance of a proposition which would be repudiated. The old stock instance will do as well as another—'Come now, sir, answer "yes" or "no." Have you left off beating your father yet[1]?' Either answer leads to an apparent admission of impiety.

A late Senior Proctor once enraged a man at a fair with this form of fallacy. The man was exhibiting a blue horse; and the distinguished stranger asked him—'With what did you paint your horse?'

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[1] D. L. II, § 135. Another form is 'Postulo uti respondeas, desierisne facere adulterium an non.' Aul. Gell. XVI. 2, § 5.

EXERCISES.

EXERCISES.

(Key sent by the author on receipt of 3s. 6d.
Address—16 Museum Road, Oxford.)

PART I.

CHAPTER I.

Classify the following words according as they are categorematic, syncategorematic, or acategorematic :—

come	peradventure	why
through	inordinately	pshaw
therefore	circumspect	puss
grand	inasmuch	stop
touch	sameness	back
cage	disconsolate	candle.

CHAPTER II.

Classify the following things according as they are substances, qualities, or relations :—

God	likeness	weight
blueness	grass	imposition
ocean	introduction	thinness
man	air	spirit
Socrates	raillery	heat
mortality	plum	fire.

CHAPTER III.

1. Give six instances each of—attributive, abstract, singular, privative, and relative terms.

2. Select from the following list of words such as are terms, and state whether they are (1) abstract or concrete, (2) singular or common, (3) univocal or equivocal:—

van	table	however
enter	decidedly	tiresome
very	butt	Solomon
infection	bluff	Czar
short	although	Caesarism
distance	elderly	Nihilist.

3. Which of the following words are abstract terms?—

quadruped	event	through
hate	desirability	thorough
fact	expressly	thoroughness
faction	wish	light
inconvenient	will	garden
inconvenience	volition	grind.

4. Refer the following terms to their proper place under each of the divisions in the scheme:—

horse	husband	London
free	lump	empty
liberty	rational	capital
impotent	reason	Capitol
impetuosity	irrationality	grave
impulsive	double	platinum.

5. Give six instances each of proper names and designations.

6. Give six instances each of connotative and non-connotative terms.

7. Give the extension and intension of—

sermon	animal	sky
clock	square	gold
sport	fish	element
bird	student	fluid
art	river	line
gas	servant	language.

CHAPTER IV.

Arrange the following terms in order of extension—carnivorous, thing, matter, mammal, organism, vertebrate, cat, substance, animal.

PART II.

CHAPTER I.

Give a name to each of the following sentences :—

(1) Oh, that I had wings like a dove!

(2) The more, the merrier.

(3) Come rest in this bosom, my own stricken deer.

(4) Is there balm in Gilead ?

(5) Hearts may be trumps.

CHAPTER II.

Analyse the following propositions into subject, copula, and predicate :—

(1) He being dead yet speaketh.

(2) There are foolish politicians.

(3) Little does he care.

(4) There is a land of pure delight.

(5) All's well that ends well.

(6) Sweet is the breath of morn.

(7) Now it came to pass that the beggar died.

(8) Who runs may read.

(9) Great is Diana of the Ephesians.

(10) Such things are.

(11) Not more than others I deserve.

(12) The day will come when Ilium's towers shall perish.

CHAPTER III.

1. Express in logical form, affixing the proper symbol :—

(1) Some swans are not white.

(2) All things are possible to them that believe.

(3) No politicians are unprincipled.

(4) Some stones float on water.

(5) The snow has melted.

(6) Eggs are edible.

(7) All kings are not wise.

(8) Moths are not butterflies.

(9) Some men are born great.

(10) Not all who are called are chosen.

(11) It is not good for man to be alone.

(12) Men of talents have been known to fail in life.

(13) 'Tis none but a madman would throw about fire.

(14) Every bullet does not kill.

(15) Amongst Unionists are Whigs.

(16) Not all truths are to be told.

(17) Not all your efforts can save him.

(18) The whale is a mammal.

(19) Cotton is grown in Cyprus.

(20) An honest man's the noblest work of God.

(21) No news is good news

(22) No friends are like old friends.

(23) Only the ignorant affect to despise knowledge.

(24) All that trust in Him shall not be ashamed.

(25) All is not gold that glitters.

(26) The sun shines upon the evil and upon the good.

(27) Not to go on is to go back.

(28) The king, minister, and general are a pretty trio.

(29) Amongst dogs are hounds.

(30) A fool is not always wrong.

(31) Alexander was magnanimous.

(32) Food is necessary to life.

(33) There are three things to be considered.

(34) By penitence the Eternal's wrath 's appeased.

(35) Money is the miser's end.

(36) Few men succeed in life.

(37) All is lost, save honour.

(38) It is mean to hit a man when he is down.

(39) Nothing but coolness could have saved him.

(40) Books are generally useful.

(41) He envies others' virtue who has none himself.

(42) Thankless are all such offices.

(43) Only doctors understand this subject.

(44) All her guesses but two were correct.

(45) All the Apostles were twelve.

(46) Gossip is seldom charitable.

(47) Better to play for nothing than work for nothing.

(48) All men have not faith.

(49) We have no king but Caesar.

2. Give six examples of indefinite propositions, and then quantify them according to their matter.

3. Compose three propositions of each of the following kinds :—

    (1) with common terms for subjects ;

    (2) with abstract terms for subjects ;

    (3) with singular terms for predicates ;

(4) with collective terms for predicates;
(5) with attributives in their subjects;
(6) with abstract terms for predicates.

CHAPTER IV.

1. Point out what terms are distributed or undistributed in the following propositions:—

(1) The Chinese are industrious.
(2) The angle in a semi-circle is a right angle.
(3) Not one of the crew survived.
(4) The weather is sometimes not propitious.

The same exercise may be performed upon any of the propositions in the preceding list.

2. Prove that in a negative proposition the predicate must be distributed.

CHAPTER V.

Affix its proper symbol to each of the following propositions:—

(1) No lover he who is not always fond.
(2) There are Irishmen and Irishmen.
(3)   Men only disagree,
    Of creatures rational.
(4) Some wise men are poor.
(5) No Popes are some fallible beings.
(6) Some step-mothers are not unjust.
(7) The most original of the Roman poets was Lucretius.
(8) Some of the immediate inferences are all the forms of conversion.
(9) quidquid honestum est, idem utile videtur, nec utile quidquam, quod non honestum. Cic. *de Off*. III, § 20.
(10) Dead languages are not the only ones worth studying.

CHAPTER VI.

1. Give six examples of terms standing one to another as genus to species.

2. To which of the heads of predicables, if any, would you refer the following statements? And why?

(1) A circle is the largest space that can be contained by one line.

(2) All the angles of a square are right angles.

(3) Man alone among animals possesses the faculty of laughter.

(4) Some fungi are poisonous.

(5) Most natives of Africa are negroes.

(6) All democracies are governments.

(7) Queen Anne is dead.

(8) A horse is the animal you saw yesterday.

(9) An honest man's the noblest work of God.

3. In what relation do these attributes stand to an isosceles triangle?

(1) that the angles at the base are equal;

(2) that the three angles are equal to two right angles.

CHAPTER VIII.

Examine the following attempts at definition both by the material and by the formal rules. If you are dissatisfied with any of them, substitute, where you can, a better definition. Point out any that seem to you to coincide in meaning.

(1) An acute-angled triangle is one which has an acute angle.

(2) An archdeacon is one who exercises archidiaconal functions.

(3) Architecture is frozen music.

(4) An attributive is a term which cannot stand as a subject.

(5)   *a.* Bread is the staff of life.

     b. Bread is food in the form of loaves.

(6) A candle is a kind of light used before gas was invented.

(7)   *a.* The cause of anything is the antecedent which it invariably follows.   Mill (III. 5, § 5).

     b. The cause of a phenomenon is the antecedent, or the concurrence of antecedents, on which it is invariably and unconditionally consequent.   *Ibid.*

     c. A cause is the assemblage of phenomena, which occurring, some other phenomenon invariably commences, or has its origin.   Mill (III. 5, § 6.)

     d. A cause is that without which something would not be.

(8) Caviare is a kind of food.

(9) A circle is a plane figure contained by one line.

(10)   *a.* A citizen is a person both of whose parents were citizens.

     b. A citizen is one who is qualified to exercise deliberative and judicial functions.   Arist. *Pol.* III. 1, § 12.

     c. A citizen is a man who pays taxes.

(11) Credit is the bond of society.

(12)   *a.* Death is the separation of the soul from the body.   Plato, *Phaedo.*

     b. Death is the extinction of the vital forces.

     c. Mors est naturae animantium dissolutio.   Lactantius.

     d. Mors est aeterni doloris perpessio.   *Ibid.*

     e. Death is the end of life.

(13) Deliberation is that species of investigation which is concerned with matters of action.

(14) *a.* A dog is an animal of the canine species.

      b. A dog is a domestic animal that barks.

      c. A dog is a wild animal that howls.

(15) An eccentricity is a peculiar idiosyncrasy.

(16) Eloquence is the power of influencing the feelings by speech or writing.

(17) Fame is a fancied life in others' breath.

(18) A fault is a quality productive of evil or inconvenience.

(19) *a.* A gentleman is a person who moves in good society.

      b. A gentleman is a man who respects himself and others.

      c. A gentleman is a person who has no visible means of subsistence. The Tichborne Claimant.

      d. A gentleman is a person who has nothing to do and does it.

      e. A gentleman is a man of gentle birth and gentle manners.

      f. A gentleman is a man of independent means.

(20) *a.* Grammar is the science of language.

      b. Grammar is a branch of philology.

      c. Grammar is the art of speaking and writing a language with propriety.

(21) *a.* Humour is thinking in jest while feeling in earnest.

      b. Humour is the perception of unexpected incongruities.

(22) *a.* Induction is the operation of discovering and proving general propositions. Mill (III. 1, § 2).

      b. Induction is experience or observation consciously looked at in a general form. Whewell.

      c. Induction is the process by which we conclude that what is true of certain individuals of a class is

true of the whole class, or that what is true at certain times will be true in similar circumstances at all times. Mill (III. 2, § 1).

d. Induction is the colligation of facts by means of appropriate conceptions. See Mill III. 2, § 4.

e. Induction is Generalisation from Experience. Mill (III. 3, § 1).

(23)   *a.* Ira furor brevis est. Horace.

b. Ira cupiditas est poenae exigendae. Seneca.

c. Ira est cupiditas puniendi eius, a quo te inique putes laesum. Posidonius (apud Lact.).

d. Ira est incitatio animi ad nocendum ei, qui aut nocuit, aut nocere voluit.

e. Ira est motus animi ad coercenda peccata insurgentis. Lactantius.

(24)   *a.* Justice is minding one's own business and not being meddlesome. Plato.

b. Justice is an inner state of the soul that sets a man at peace with himself and the world.

c. Justice is that sort of state in consequence of which men are able to do what is just, and in consequence of which they deal justly, and wish for what is just. Arist. *E. N.* v. 1, § 3.

d. Justice is telling the truth and restoring what you have taken.

e. Justice is rendering to each his due. Simonides.

f. Justice is doing good to one's friends and harm to one's foes.

g. Justice is the interest of the stronger.

(25) Length is that dimension of a solid which would be measured by the longest line.

(26)   *a.* Life is bottled sunshine. Winwood Reade.

b. Life is the opposite of death.

c. Life is the definite combination of heterogeneous changes, both simultaneous and successive, in correspondence with external coexistences and sequences. Herbert Spencer.

(27)   *a.* Logic is the science of the formal laws of thought. Sir William Hamilton.

b. Logic may be considered as the science and also as the art of reasoning.   Whately.

c. Logic is the science which treats of the operations of the human understanding in the pursuit of truth.

d. Logic is the science of the operations of the understanding which are subservient to the estimation of evidence: both the process itself of advancing from known truths to unknown, and all other intellectual operations in so far as auxiliary to this. Mill (*Introd.* § 7).

e. Logic is the science of proof or evidence.

f. Logic is the entire theory of the ascertainment of reasoned or inferred truth.

g. Logic is the 'science of the Investigation of Truth by means of Evidence.'   Mill (III. 5, § 2).

h. Logic is the Art of Thinking, which means of correct thinking, and the Science of the Conditions of correct thinking.   Mill (*Exam. of Sir Wm. H.*, 3rd ed. p. 448).

i. La logique est l'art de bien conduire sa raison dans la connaissance des choses, tant pour s'instruire soi-même, que pour en instruire les autres. *Logique de Port Royal.*

j. Logic is the science of the conditions on which correct thoughts depend, and the art of attaining to correct and avoiding incorrect thoughts.   Fowler.

k. Logic is the science of argument, i.e. of inference and proof.   *Palaestra Logica.*

(28)   *a.* Love is the opposite of hatred.

     b. Love is the fulfilling of the law.

     c. Love is the union of hearts.

(29)   *a.* Man is an animal that makes exchanges.

     b. Man is a rational biped.

     c. Man is a religious animal.

     d. Man is a self-conscious rational mind-entity, involved in body.   Laurie.

     e. Man is any being that is born of human parents.

     f. Man is an animal that expresses general ideas by means of signs.

(30) Necessity is the mother of invention.

(31) Nec-manifestum furtum quid sit, ex iis quae diximus intellegitur; nam quod manifestum non est, id scilicet nec-manifestum est.   Justinian (*Inst.* IV. 1, 3).

(32) A net is a collection of holes strung together.

(33) Noon is the time when the shadows of bodies are shortest.

(34) North is the direction in which we look towards the position of the sun at midnight.

(35) An oligarchy is the supremacy of the rich in a state. Arist. *Pol.*

(36) A parable is a heavenly story with no earthly meaning.

(37) A parallelogram is a four-sided figure, having its opposite sides parallel and equal.

(38) Peace is the absence of war.

(39)   *a.* Philosophy is the rule of life.

     b. Philosophy is the attempt of man to ascertain his relations to God, to the universe, and to his fellow-creatures.

     c. Philosophia nihil aliud est quam recta vivendi ratio.   Seneca.

(40) A plant is an organised being possessing vegetable life.

(41) Politeness is the oil that lubricates the wheels of society.

(42) Prudence is the ballast of the moral vessel.

(43) *a.* The ridiculous is some fault or ugliness, unaccompanied with pain, and not tending to destruction. Arist. *Poet.*

   b. The ridiculous is that which gives you a sense of superiority.

(44) Rust is the red desquamation of old iron.

(45) The sun is the centre of the solar system.

(46) Sense is the recognition, adjustment and maintenance of the proper and fitting relations of the affairs of ordinary life.

(47) Superstition is a tendency to look for constancy where constancy is not to be expected.

(48) A tip is an extra gratuity paid out of good-will, over and above what can be demanded by contract.

(49) *a.* Virtue is the capacity of ruling over men. Plato, *Meno.*

   b. Virtue is to desire noble things and be able to attain them. *Ibid.*

   c. Virtue is the procuring good things justly. *Ibid.*

   d. Virtue is acting virtuously.

   e. Virtue is that line of conduct which tends to produce happiness.

   f. Virtue is the preference for the desire which is felt to be higher over that which is felt to be lower.

   g. Virtus est vitium fugere. Horace.

   h. Virtus est malis ac vitiis fortiter repugnare. Lactantius.

   i. Virtus est iram cohibere, cupiditatem compescere, libidinem refraenare. Lactantius.

   j. Virtue is the will to do right.

k. Virtue is the control of the feelings and actions by reason.

(50) *a.* Wealth is the sum of the necessaries and conveniences of life.

b. Wealth is all useful or agreeable things which possess exchangeable value. Mill.

c. Wealth is the possession of the valuable by the valiant. Ruskin.

d. Wealth is the material means to happiness.

e. Wealth is any material product which is held to contribute to human happiness.

CHAPTER IX.

Criticise the following as divisions—

(1) Books into octavo, quarto, green, and blue.
(2) Chair into—

    a. seat, back, legs, arms.

    b. arm-chair, rocking-chair, cane-bottomed chair, wooden chair.

(3) Church into Gothic, episcopal, high, and low.
(4) Ends into those which are ends only, means and ends, and means only.
(5) Figure into curvilinear and rectilinear.
(6) Great Britain into England, Scotland, Wales, and Ireland.
(7) Horses into race-horses, hunters, hacks, thoroughbreds, ponies, and mules.
(8) Library into public and private.
(9) Pictures into sacred, historical, landscape, and mythological.
(10) Plant into stem, root, and branches.
(11) Science into physical, moral, metaphysical, and medical.

(12) Ship into frigate, brig, schooner, and merchant-man.

(13) Thing into good, bad, and indifferent.

(14) Triangle into—

    a. acute-angled, right-angled, and obtuse-angled.

    b. equilateral, isosceles, and scalene.

(15) Vertebrate animals into quadrupeds, birds, fishes, and reptiles.

(16) Warship into battle-ship, cruiser, coast defence vessel, torpedo-boat, and destroyer.

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PART III.

CHAPTER V.

What principles are referred to here—

(1) Nec eventus modo hoc docuit (stultorum iste magister est), sed eadem ratio, quae fuit futuraque, donec res eaedem manebunt, immutabilis est. Liv. XXII. 39, § 10.

(2) Agitur de parricidio, quod sine multis causis suscipi non potest ; apud homines autem prudentissimos agitur, qui intellegunt neminem ne minimum quidem maleficium sine causa admittere. Cic. *Pro Rosc. Am.* § 73.

(3) Nil igitur fieri de nilo posse fatendumst. Lucr. I. 205.

CHAPTER IX.

1. What inductive methods, if any, are employed in the following examples?

(1) I own myself entirely satisfied . . . that there is no such thing as colour really inhering in external bodies, but that it is altogether in the light. And what confirms me in this opinion is that in propor-

tion to the light colours are still more or less vivid ; and if there be no light, then are there no colours perceived. Berkeley (Fraser's edit. Vol. I, p. 277).

(2) Wealth causes Christianity, for the wealthiest nations are Christian.

(3) He said he had always throughout Kerry found that wherever there was a local branch of the National League actively working there were also Moonlighters ; and he believed they were connected. *Standard*, Nov. 30, 1888.

(4) Professor Zdekauer, the first authority in Russia, said he had witnessed five epidemics of cholera, each of which was preceded by an epidemic of influenza, such as that now raging. He considered it highly probable that the present disease would be succeeded by cholera next spring. *Standard*, Dec. 2, 1889.

(5) The increase of agrarian crime, say the Judges, was coincident with the activity of the Land League, and the decrease of agrarian crime with the inactivity of the Land League. *Standard*, Feb. 14, 1890.

(6) In reply to my question about the porpoise-grease with which his body was anointed, Captain Webb informed me that he did not know that it helped him at all . . . However of course this point could only be settled by the direct experience of a swimmer performing a feat under exactly similar conditions both with and without the oil. *Daily News*, Sept. 17, 1875.

2. What principle is appealed to here ?—

'A different cause,' says Doctor Sly,
  'The same effect may give :
Poor Lubin weeps lest he should die,
  His wife, lest he should live.'

PRIOR.

CHAPTER XI.

1. Found arguments upon the following analogies—

    (1) As health : the body : : virtue : the soul.

    (2) As a picture : painting : : a law : statesmanship.

    (3) As the eye : the body : : Athens : Greece.

    (4) As the size of the universe : that of Socrates : : its power, wisdom, and thought : that of Socrates.

    (5) As fallacies : logic : : the doctrine of false notions : the interpretation of nature.

    (6) As the Church : Christ : : wives : their husbands.

    (7) As speech : reason : : the sensible world : the intellectual world.

    (8) As assertion and denial : the intellect : : pursuit and avoidance : the emotions.

2. Show how the following lend themselves to metaphor—

    (1) As the bowl : Bacchus : : the shield : Ares.

    (2) As old age : life : : evening : day.

3. Criticise the following—

    tempore (ut fluvio) leviora et magis inflata ad nos devehente, graviora et solida mergente.　Bacon.

4. Exhibit the analogy that underlies these metaphors—

The great question which is now before the United Kingdom might be called Local Option in Government, just as the Bill Sir William Harcourt is to introduce to-day might be described as Home Rule in Drink. *Daily Chronicle*, Feb. 27, 1893.

5. Examine the following argument—

Architecture is to building what literature is to language.
    No nation is without some kind of literature.
    ∴ No nation is without some kind of architecture.

CHAPTER XIII.

1. Give the logical opposites of the following propositions—

    (1) Knowledge is never useless.

    (2) All Europeans are civilised.

    (3) Some monks are not illiterate.

    (4) Happy is the man that findeth wisdom.

    (5) No material substances are devoid of weight.

    (6) Every mistake is not culpable.

    (7) Some Irishmen are phlegmatic.

2. Granting the truth of the following propositions, what other propositions can be inferred by opposition to be true or false?

    (1) Men of science are often mistaken.

    (2) He can't be wrong, whose life is in the right.

    (3) Sir Walter Scott was the author of *Waverley*.

    (4) The soul that sinneth it shall die.

    (5) All women are not vain.

3. Granting the falsity of the following propositions, what other propositions can be inferred by opposition to be true or false?—

    (1) Some men are not mortal.

    (2) Air has no weight.

    (3) All actors are improper characters.

    (4) None but dead languages are worth studying.

    (5) Some elements are compound.

4. Examine this argument—

    Now if Christ is preached that he hath been raised from the dead, how say some among you that there is no resurrection of the dead? But if there is no resurrection of the dead, neither hath Christ been raised. 1 Cor. xv. 12, 13.

5. Explain and illustrate these statements—

To establish the universal is more constructive of your own position than to establish the particular.

To refute the particular is more destructive of your opponent's position than to refute the universal.

6.   All statesmen are dishonest.
            This statesman is not dishonest.

Can the above propositions

(1) be both true,
(2) be both false?

What name would you give to them in relation to one another?

7. Why do we derive more information from refuting a particular than from refuting a universal?

8. Why is the opposition between sub-contraries apparent, not real?

CHAPTER XIV.

1. Give, as far as possible, the logical converse of each of the following propositions—

(1) Energy commands success.
(2) Mortals cannot be happy.
(3) There are mistakes which are criminal.
(4) All's well that ends well.
(5) Envious men are disliked.
(6) A term is a kind of word or collection of words.
(7) Some Frenchmen are not vivacious.
(8) All things in heaven and earth were hateful to him.
(9) The square of three is nine.
(10) All cannot receive this saying.
(11) The magic of property turns sand into gold.

(12) He who fights and runs away
     Will live to fight another day.
(13) I am what I am.
(14) Some dogs are larger than some ponies.
(15) Someone has blundered.
(16) P struck Q.
(17) Amas.

2. In the following passages is the use of logical language correct?

(1) More things may be contained in my philosophy than
    exist in heaven or earth: but the converse pro-
    position is by no means true.

(2) If Mr. Chamberlain had learned logic, he would be
    aware that a proposition does not imply its converse.
    Being a practical man, he must know that he is
    talking nonsense. From the perfectly gratuitous
    assumption that the Liberals will not be able to
    disestablish the Church in Wales he draws the wholly
    erroneous inference that the Tories are both able
    and willing to do so. *Daily News*, Jan. 1, 1892.

(3) That great wits are to madness near allied has become
    a proverb consolatory to the world. Unluckily the
    world forgets that the proposition, even if true, is not
    ‘convertible,’ as logicians say. Genius may be akin to
    madness, but it does not follow that madness, in all
    its shapes, is akin to genius. *Daily News*, May 23,
    1891.

(4) The Democrats themselves admitted that, if they
    could not carry the Empire State, there would be
    a Republican President. They have carried it, and,
    though a proposition does not necessarily imply its
    converse, their hopes will of course be immensely
    raised. *Daily News*, Nov. 5, 1891.

CHAPTER XV.

Obvert the following propositions—

- (1) All just acts are expedient.
- (2) No display of passion is politic.
- (3) Some clever people are not prudent.
- (4) Some philosophers have been slaves.

The same exercise may be performed upon any of the propositions in the preceding lists.

CHAPTER XVI.

1. Give the converse by negation of—

- (1) All women are lovely.
- (2) Some statesmen are not practical.
- (3) All lawyers are honest.
- (4) All doctors are skilful.
- (5) Some men are not rational.
- (6) Some tyrants have not been unprosperous.

2. Give the converse by contraposition of—

- (1) All solid substances are material.
- (2) All the men who do not row play cricket.
- (3) All impeccable beings are other than human.
- (4) Some prejudiced persons are not dishonest.
- (5) All the pieces that are not white are red.
- (6) All the pieces that are white are not red.
- (7) No A is not-A.
- (8) Some wholesome things are not pleasant.
- (9) All metals are elements.
- (10) No good men are insincere.
- (11) Some sandy soils are not unfertile.

3. Prove indirectly the truth of the contrapositive of
   All A is B.

4. Criticise the following as immediate inferences—

 (1) All wise men are modest.
  ∴ No immodest men are wise.

 (2) Some German students are not industrious.
  ∴ Some industrious students are not Germans.

 (3) Absolute difference excludes all likeness.
  ∴ Any likeness is a proof of sameness.

 (4) None but the brave deserve the fair.
  ∴ All brave men deserve the fair.

 (5) All discontented men are unhappy.
  ∴ No contented men are unhappy.

 (6) Books being a source of instruction, our knowledge must come from our libraries.

 (7) All Jews are Semitic.
  ∴ Some non-Semitic people are not Jews.

 (8) Wherever the kitten is, the cat is.
  ∴ Wherever the cat is, the kitten is.

 (9) None but metaphysicians understand Hegel.
  ∴ Some metaphysicians understand Hegel.

 (10) All the equilateral triangles are all the equiangular.
  ∴ Any triangle which is not equilateral is not equi-angular.

 (11) All wise men are cautious.
  ∴ No unwise men are incautious.

 (12) 'Anima,' inquit, 'quae peccaverit, ipsa morietur.'
  Ergo quae non peccaverit, ipsa vivet.
       (St. Jerome *Epist.* IX. § 8. Hurter.)

5. Show by what kind of inference each of the subjoined propositions follows from

    All discontented men are unhappy.

 (1) All happy men are contented.
 (2) Some discontented men are unhappy.

(3) Some contented men are happy.

(4) Some unhappy men are not contented.

(5) No discontented men are happy.

(6) Some happy men are contented.

(7) Some contented men are not unhappy.

(8) Some unhappy men are discontented.

(9) No happy men are discontented.

(10) Some discontented men are not happy.

(11) Some happy men are not discontented.

(12) None but unhappy men are discontented.

From how many of these propositions can the original one be derived? And why not from all?

6. Why does conversion by contraposition only apply by limitation to E?

Why does it not apply at all to I? Illustrate this by means of the proposition 'Some things are substances.'

7. From All not-x is y does it follow that

(1) Some x is y,

(2) Some x is not y?

Show the relation of these two propositions to the original.

8. Show that the contradictory of E is the same as the converse of the contradictory of its converse by limitation.

CHAPTER XVII.

1. What kind of inference have we here?—

(1)   None but the ignorant despise knowledge.

∴ No wise man despises knowledge.

(2)   A is superior to B.

∴ B is inferior to A.

2. Draw inferences from the following propositions—

Philip was the father of Alexander.

The Roman *as*, in the later ages of the Republic, was reduced to the twenty-fourth part of its original value.

CHAPTER XVIII.

1. Convert the following propositions—

    (1) If a man is wise, he is humble.

    (2) Where there is sincerity, there is no affectation.

    (3) When night-dogs run, all sorts of deer are chased.

    (4) The nearer the Church, the further from God.

    (5) If there were no void, all would be solid.

    (6) Not to go on is sometimes to go back.

2. Express in a single proposition—

If he was divine, he was not covetous; and if he was covetous, he was not divine.

3. Exhibit the exact logical relation to one another of the following pairs of propositions—

    (1) If the conclusion be false, the premisses are false.

        If the conclusion be true, the premisses are not necessarily true.

    (2) If one premiss be negative, the conclusion must be negative.

        If the conclusion be negative, one of the premisses must be negative.

    (3) The truth of the universal involves the truth of the particular.

        The falsity of the particular involves the falsity of the universal.

    (4) From the truth of the particular no conclusion follows as to the universal.

        From the falsity of the universal no conclusion follows as to the particular.

    (5) If the conclusion in the fourth figure be negative, the major premiss must be universal.

        If the major premiss in the fourth figure be particular, the conclusion must be affirmative.

(6) If both premisses be affirmative, the conclusion must be affirmative.

If the conclusion be negative, one of the premisses must be negative.

(7) Who once has doubted never quite believes.

Who once believed will never wholly doubt.

4. 'The Method of Agreement stands on the ground that whatever circumstance can be eliminated is not connected with the phenomenon by any law; the Method of Difference stands on the ground that whatever circumstance cannot be eliminated is connected with the phenomenon by a law.' Do these two principles imply one another?

CHAPTER XIX.

Fill up the following enthymemes, mentioning to which order they belong, and state which of them are expressed in problematic form—

(1) I am fond of music; for I always like a comic song.

(2) All men are born to suffering, and therefore you must expect your share.

(3) Job must have committed some secret sins: for he fell into dreadful misfortunes.

(4) Latin was the language of the Vestals, and therefore no lady need be ashamed of speaking it.

(5) None but physicians came to the meeting. There were therefore no nurses there.

(6) The human soul extends through the whole body, for it is found in every member.

(7) No traitor can be trusted, and you are a traitor.

(8) Whatever has no parts does not perish by the dissolution of its parts. Therefore the soul of man is imperishable.

(9)   The Christ had to suffer and to rise from the dead.
       This Jesus, whom I preach unto you, is the
       Christ.   Acts xvii. 3.

(10)   Both health and wealth may be used well or ill.
       ∴ Neither health nor wealth is a good in itself.

Is the suppressed premiss in any case disputable on material
grounds?

CHAPTERS XX—XXVIII.

I.

Refer the following arguments to their proper mood and
figure, or show what rules of syllogism they violate—

(1)   No miser is a true friend, for he does not assist
       his friend with his purse.

(2)   Governments are good which promote prosperity.
       The government of Burmah does not promote
       prosperity.
       ∴ It is not a good government.

(3)   Men are sinners.
       Saints are men.
       ∴ Saints are sinners.

(4)   Nothing is property but that which is the product
       of man's hand.
       The horse is not the product of man's hand.
       ∴ The horse is not property.

(5)   Some Europeans at least are not Aryans, because
       the Finns are not.

(6)   Saturn is visible from the earth, and the moon is
       visible from the earth.   Therefore the moon is
       visible from Saturn.

(7)   Some men of self-command are poor, and there-
       fore some noble characters are poor.

(8) Sparing the rod spoils the child : so John will turn out very good, for his mother beats him every day.

(9) Some effects of labour are not painful, since every virtue is an effect of labour.

(10) The courageous are confident and the experienced are confident. Therefore the experienced are courageous.

(11) No tale-bearer is to be trusted, and therefore no great talker is to be trusted, for all tale-bearers are great talkers.

(12) Socrates was wise, and wise men alone are happy : therefore Socrates was happy.

(13) Malum est avaritia ; multos enim magnis incommodis adfecit pecuniae cupiditas. Cic. *De Inv.* I, § 95.

(14) Half a loaf is better than no bread.
No bread is better than no friends.
∴ Half a loaf is better than no friends.

(15) Whatever is used as the medium of exchange is money.
Cattle are money.
∴ Cattle are used as the medium of exchange.

(16) No joke is always in season.
An examination is no joke.
∴ An examination is always in season.

II.

1. From the major 'No matter thinks' draw, by supplying the minor, the following conclusions—

   (1) Some part of man does not think.
   (2) The soul of man is not matter.
   (3) Some part of man is not matter.
   (4) Some substance does not think.

Name the figured mood into which each syllogism falls.

2. Construct syllogisms in the following moods and figures, stating whether they are valid or invalid, and giving your reasons in each case—

> AEE in the first figure; EAO in the second; IAI in the third; AII in the fourth.

3. Prove that ' Brass is not a metal,' using as your middle term 'compound body.'

4. Construct syllogisms to prove or disprove—

> (1) Some taxes are necessary.
> (2) No men are free.
> (3) Laws are salutary.

5. Prove by a syllogism in Bocardo that ' Some Socialists are not unselfish,' and reduce your syllogism directly and indirectly.

6. Prove the following propositions in the second figure, and reduce the syllogisms you use to the first—

> (1) All negroes are not averse to education.
> (2) Only murderers should be hanged.

7. Prove in Baroco and also in Ferio that ' Some Irishmen are not Celts.'

8. Construct in words the same syllogism in all the four figures.

9. Invent instances to show that false premisses may give true conclusions.

III.

1. What moods are peculiar to the first, second, and third figures respectively?

2. What moods are common to all the figures?

3. Why can there be no subaltern moods in the third figure?

4. What is the only kind of conclusion that can be drawn in all the figures?

5. Show that IEO violates the special rules of all the figures.

6. In what figures is AEE valid?

7. Show that AEO is superfluous in any figure.

8. Prove that O cannot be a premiss in the first figure, nor a minor premiss anywhere but in the second.

9. Show that in the first figure the conclusion must have the quality of the major premiss and the quantity of the minor.

10. Why do the premisses EA yield a universal conclusion in the first two figures and only a particular one in the last two?

11. Show that, if the major term be distributed in the premiss and undistributed in the conclusion, the mood must be AAI.

IV.

1. Why is it enough to distribute the middle term once?

2. What is the least number of terms that can be distributed in the premisses of a syllogism?

3. What is the greatest number of terms that can be distributed in the premisses of a syllogism?

4. Why must there be at least one more term distributed in the premisses than in the conclusion?

5. Prove that the number of distributed terms in the premisses cannot exceed those in the conclusion by more than two.

6. Prove that the number of undistributed terms in the premisses cannot exceed those in the conclusion by more than one.

7. Prove that wherever the minor premiss is negative the major must be universal.

8. Prove that wherever the minor term is distributed the major premiss must be universal.

9. If the middle term be twice distributed, what mood and figure are possible?

10. When the middle term is distributed in both premisses, what must be the quantity of the conclusion?

11. If the major term of a syllogism be the predicate of the major premiss, what do we know about the minor premiss?

12. Prove that, if the conclusion be universal, the middle term can only be distributed once in the premisses.

V.

1. Examine the following arguments in accordance with the special rules of the four figures. If necessary, restate them.

    (1)   Some one like me has come.
           None but Orestes is like me.
      ∴ Orestes has come.

    (2)   Their syllogism runs somewhat like this.
           France and the United States are Republics;
           they have both shown strong tendencies to corruption;
           therefore Republics are liable to be corrupt.

It would be interesting to lay such a syllogism before a professor of logic, and ask him what he thinks of it, and how many marks it would be likely to score in an examination. *The Daily Chronicle*, Dec. 23, 1892.

    (3)   No one who allows evil is good.
           God allows evil.
      ∴ God is not good.

2. Why is it that in the second figure there must always be two terms distributed?

3. In what figures can a conclusion be drawn when only one term is distributed in the premisses?

4. In what moods of the different figures are there three terms distributed in the premisses?

5. Prove that in the fourth figure—

    (1) When the minor premiss is particular, the major must be negative.

    (2) When the minor premiss is negative, both premisses must be universal.

    (3) When the conclusion is negative, the major premiss must be universal.

    (4) The conclusion cannot be a universal affirmative.

    (5) Neither of the premisses can be a particular negative.

    (6) When the major premiss is particular, the conclusion must be affirmative.

Show what relation (1) bears to the first rule of Figure IV, and what relation (3) and (6) bear to one another.

6. If All P and no S is M, show that Some S is not P and Some P is not S.

7. Is the conclusion here wider than the premisses?—

        Job was patient.

        Job was a man.

    ∴ Some men are patient.

VI.

1. To what moods only of the first figure are those of the second directly reducible by the ordinary method? And why?

2. To what moods only of the first figure are those of the third directly reducible? And why?

3. In the ordinary mnemonic lines when there is m in the name of a figured mood, there is always a consonant at the end. Why is this?

4. Why are the premisses of **Fesapo** and **Fresison** not transposed in reduction like those of the other moods of the fourth figure?

5. In what sense is it possible to reduce a particular mood to a universal?

6. Prove that in the indirect reduction of the first to the second figure the minor premiss cannot be retained in the negative moods.

7. Why cannot **Ferio** be reduced indirectly to the third figure by retaining the major premiss?

8. Prove that **Baroco** cannot be reduced indirectly to the fourth figure.

9. Prove that no other mood in the second figure can be indirectly reduced to **Camestros**.

10. Why cannot **Ferio** be indirectly reduced to **Barbari**?

CHAPTER XXIX.

1. Show by reduction that this is valid reasoning—

> If C is not D, E is not F.
> If A is not B, C is not D.
> ∴ If A is not B, E is not F.

2. Examine the following arguments—

(1) If Dion is a horse, Dion is an animal.
But Dion is not a horse.
∴ Dion is not an animal.

(2) If you have faith you can remove mountains.
But the mountains are not removed.

(3) If a thing must be, it can be. But, if it can, it also cannot be. Therefore, if a thing must be, it cannot be.

3. With the following major construct as many conjunctive syllogisms as you can—

> If Homer wrote the Iliad, he was the greatest poet of antiquity.

4. Reduce to logical form and supply a concrete instance of the following reasoning—

> If C is not D, A is B.
> If A is not B, E is F.
> ∴ If E is F, C is sometimes D.

5. Invent a concrete instance of the following kind of reasoning—

> If A is B, either C is D or E is F.
> C is not D.
> ∴ If A is B,   E is F.

CHAPTER XXXI.

1. Assign their proper place to these two arguments and invent concrete instances—

> (1) A is either B or C.   (2) B is either C or D.
>   A is not B.   A is B.
> ∴ A is C.   ∴ A is either C or D.

2. Granted that everything is either x or y, is it still possible for some x to be y?

3. Fill up these enthymemes—

> (1) Zeno is not mistaken.
> ∴ Some cobbler is a king.

> (2) Either Jesus was mad or bad or else we must believe him.

4. Examine the following—

> (1) Unum quidem certe, nemo erit tam iniquus Cluentio, qui mihi non concedat, si constet corruptum illud

esse iudicium, aut ab Avito aut ab Oppianico esse
corruptum. Si doceo non ab Avito, vinco ab Oppi-
anico; si ostendo ab Oppianico, purgo Avitum. Cic.
Pro Clu. § 64.

(2) Quoniam habes istum equum, aut emeris oportet aut
hereditate possideas aut munere acceperis aut domi
tibi natus sit aut, si horum nihil est, surripueris
necesse est : sed neque emisti neque hereditate venit
neque donatus est neque domi natus est ; necesse
est ergo surripueris.   Cic. *De Inv.* I, § 84.

CHAPTERS XXIX—XXXII.

1. Fill up the following enthymemes, and state the exact
nature of the resulting syllogism—

(1)   If Livy is a faultless historian, we must believe all
that he tells us : but that it is impossible to do.

(2)   If they stay abroad, the wife will die ; while the
husband's lungs will not stand the English
climate.   It is to be feared therefore that one
must fall a victim.

(3)   He is either very good, very bad, or commonplace.
But he is not very good.

(4)   Either a slave is capable of virtue or he is not.
∴. Either he ought not to be a slave or he is not
a man.

(5)   Does not his feebleness of character indicate either
a bad training or a natural imbecility ?

(6)   Those who ask shan't have ; those who don't ask
don't want.

(7)   If a man be mad, he deviates from the common
standard of intellect.
∴. If all men be alike mad, no one is mad.

(8) ' I cannot dig ; to beg I am ashamed.'

(9) If I go on swimming, I shall cut my throat ; and if I stop swimming, I shall be drowned.

(10) If we are to be friends with the king, we shall be more useful to him, if we keep our arms, and, if we are to fight with him, we shall fight better, if we keep our arms.   Xen. *Anab.* II. 1, § 20.

(11) I cannot go on this tour : for, if I do, I must ride a bicycle and wear a great-coat.

(12) Either John does not respect his father, or he does not love him.
     ∴ Either John is not wise or he is not amiable.

(13) If God were good, he would will to destroy evil, and, if he were almighty, he would be able.

(14) If you give to a beggar, you feel a fool ; and, if you refuse to give to him, you feel a beast.

(15) A newspaper is either truthful or untruthful.
     ∴ Either believe your daily paper or give up taking it in.

2. ' The infinite divisibility of space implies that of time. If the latter therefore be impossible, the former must be equally so.'   Formulate this argument as an immediate inference.

3. Examine the following arguments and refer them to their proper head—

(1) If we have a dusty spring, there is always a good wheat harvest. We shall therefore have a poor harvest this year, for the spring has not been dusty.

(2) Virtues are either feelings, capacities, or states ; and as they are neither feelings nor capacities, they must be states.

(3)   Everything must be either just or unjust.
     Justice is a thing, and is not unjust.

  ∴ Justice is just.
     Similarly holiness is holy.

     But the virtues of knowledge, justice, courage, temperance, and holiness, were declared to be different from one another.

  ∴ Justice is unholy and holiness unjust.

(4)   If he observes the sabbath or if he refuses to eat pork, he is a Jew.
     But he both observes the sabbath and refuses to eat pork.

  ∴ He is a Jew.

(5)   If this triangle is equilateral, its sides and its angles will be equal.
     But neither its sides nor its angles are equal.

  ∴ It is not equilateral.

(6)   If the barometer falls, there will be either wind or rain.
     There is neither wind nor rain.

  ∴ The barometer has not fallen.

4. Rebut the following dilemmas—

  (1)   If I tell the truth, I shall offend the people; and if I tell a lie, I shall offend my conscience.
       Either I must tell the truth or tell a lie.

    ∴ Either I shall offend the people or offend my conscience.

  (2)   If he is sensible to shame, he ought not to be scolded, and if he is not, he won't mind it.

  (3)   If I don't wear my gown, I shall be fined by the Proctors and, if I do, I shall be laughed at by the men.

CHAPTER XXXIII.

1. Formulate the following trains of reasoning, resolve them
into their component parts, and point out any violations of
the rules of syllogism which they may contain—

(1) No Church Institutions are useful ; for they teach
religious matters, not business matters, which
latter are useful, being profitable.

(2) Mr. Darwin long ago taught us that the clover crop
is dependent on the number of maiden ladies in
the district. For the ladies keep cats, and the
cats destroy the field-mice, which prey on the
bees, which, in their turn, are all-important agents
in the fertilisation of the clover flowers.

(3) Athletic games are duties ; for whatever is neces-
sary to health is a duty, and exercise is necessary
to health, and these games are exercise.

(4) The iron-trade leads to the improvement of a new
country ; for furnaces require to be fed with fuel,
which causes land to be cleared.

(5) ' Is a stone a body ?' ' Yes.' ' Well, is not an animal
a body ?' ' Yes.' ' And are you an animal ?' ' It
seems so.' ' Then you are a stone, being an
animal.'

(6) If A is B, C is D.
   If E is F, G is H.
   But if A is B, E is F.
   ∴ If C is D, G is sometimes H.

(7) All good things are fair.
   Eros is without the fair, else he would not desire it.
   ∴ Eros is without the good.

Plat. *Symp.* 201 c.

(8) All men are fallible, for they are finite.
   All Popes are men, for they are born of women.

(9) Illud autem perabsurdum, bonum esse aliquid, quod non expetendum sit, aut expetendum quod non placens, aut, si id, non etiam diligendum ; ergo etiam probandum : ita etiam laudabile : id autem honestum. Ita fit ut, quod bonum sit, id etiam honestum sit. Cic. *Fin.* III, § 27.

(10) Si indigetis pecuniae, pecuniam non habetis ; si pecuniam non habetis, pauperes estis : indigetis autem pecuniae ; mercaturae enim, ni ita esset, operam non daretis ; pauperes igitur estis. Cic. *De Inv.* I, § 88.

(11) The principles which all mankind allow for true are innate ; those that men of right reason admit are principles allowed by all mankind ; we and those of our mind are men of reason ; therefore, we agreeing, our principles are innate. Locke, *Essay* I. 3, § 20.

(12) Why, if thou never wast at court, thou never sawest good manners ; if thou never sawest good manners, then thy manners must be wicked ; and wickedness is sin, and sin is damnation. Thou art in a parlous state, shepherd. Shakespeare, *As You Like It*, Act III, sc. 2.

2. In Adam Smith, *Wealth of Nations*, Bk. IV, ch. 2, there is some reasoning which may be stated thus—

Nothing but augmentation of capital increases industry.
Nothing but saving out of revenue augments capital.
∴ Nothing but saving out of revenue increases industry.

Restrictions on importation diminish revenue.
∴ Restrictions on importation are adverse to the increase of industry.

Is this valid ? If so, formulate it syllogistically.

3. Resolve the following into their component syllogisms—

(1)
All A is B.
All C is A.
All D is C.
All E is D.
∴ All E is B.

(2)
If A is not B, C is not D.
If C is not D, E is not F.
If E is not F, G is not H.
If G is not H, K is not L.
∴ If A is not B, K is not L.

(See *Lectures in the Lyceum*, p. 339.)

CHAPTER XXXIV.

1. Point out any ambiguities which underlie the following propositions—

(1) Every one who has read the book in French will recommend those who have not to read it in English.

(2) I will not do this because he did it.

(3) These are all my books.

(4) By an old statute of the date of Edward III it was accorded 'that Parliament should be holden every year once or more often if need be.'

(5) They found Mary and Joseph and the babe lying in a manger.

(6) The king and his minister are feeble and un-scrupulous.

(7) Heres meus uxori meae triginta pondo vasorum argenteorum dato, quae volet.

2. Examine the following arguments, formulating them when sound, and referring them, when unsound, to the proper head of fallacy—

(1)   We know that thou art a teacher come from God ; for no man can do these signs that thou doest, except God be with him. S. John iii. 2.

(2)   'Sir Walter Scott's novels have ceased to be popular.' 'Well, that's only because nobody reads them.'

(3)   What we produce is property.
       The sheriff produces a prisoner.
    ∴ A prisoner is property.

(4)   As all metals are not necessarily solid, we may expect some metals to be liquid.

(5)   Moses was the son of Pharaoh's daughter.
    ∴ Moses was the daughter of Pharaoh's son.

(6)   If Aeschines took part in the public rejoicings over the success of my policy, he is inconsistent in condemning it now ; if he did not, he was a traitor then.

(7)   It is wrong to stick knives into people.
    ∴ Surgeons ought to be punished.

(8)   If a thing admits of being taught, there must be both teachers and learners of it.
    ∴ If there are neither teachers nor learners of a thing, that thing does not admit of being taught.

(9)   It is unnecessary to lend books, if they are common, and wrong to lend them, if they are rare. Therefore books should not be lent from public libraries.

(10)   Seeing is believing.
    ∴ What is not seen cannot be believed.

(11)   St. Paul was not of Jewish blood, for he was a Roman citizen.

(12)   To call you an animal is to speak the truth.
        To call you an ass is to call you an animal.
    ∴ To call you an ass is to speak the truth.

(13)   Pain chastens folly. A life of ease must therefore
        be one of folly incurable.

(14)   We cannot be happy in this world; for we must
        either indulge our passions or combat them.

(15)   It must be clear to the most unlettered mind that,
        as all things were originally created by the Deity,
        including the hair on our heads and the beards on
        our faces, there can be no such thing as property.

(16)   The crime was committed by the criminal.
        The criminal was committed by the magistrate.
    ∴ The crime was committed by the magistrate.

(17)   General councils are as likely to err as the fallible
        men of whom they consist.

(18)   Dead dogs are heavier than living ones, because
        vitality is buoyant.

(19)   Deliberation is concerned with actions.
        Actions are means.
    ∴ Deliberation is concerned with means.

(20)   'No beast so fierce but has a touch of pity;
        But I have none: therefore I am no beast.'

(21)   Practical pursuits are better than theoretical.
    ∴ Mathematics are better than logic.

(22)   Death must be a good. For either the soul, ceasing
        to be, ceases to suffer, or, continuing to be, lives
        in a better state.

(23)   What is right should be enforced by law.
    ∴ Charity should be so enforced.

(24)   All animals were in the Ark.
    ∴ No animals perished in the Flood.

(25)   If he robs, he is not honourable.
        If he pays all his dues, he does not rob.
    ∴ If he pays all his dues, he is honourable.

(26) A dove can fly a mile in a minute.
    A swallow can fly faster than a dove.
  ∴ A swallow can fly more than a mile in a minute.

(27) 'I must soap myself, because it's Sunday.'
    'Then do you only soap yourself, on Sunday?'

(28) If the charge is false, the author of it is either
    ignorant or malicious.   But the charge is true.
    Therefore he is neither.

(29) All the angles of a triangle are equal to two right
    angles.
    The angle at the vertex is an angle of a triangle.
  ∴ It is equal to two right angles.

(30) Si gravis est dolor, brevis est ; si longus, levis.
    Ergo fortiter ferendus.

(31) You are not what I am.
    I am a man.
  ∴ You are not a man.

(32) The extension of the franchise is necessary, for it
    is imperative that the right of voting should be
    granted to classes who have hitherto not pos-
    sessed this privilege.

(33) If Hannibal is really victorious, he does not need
    supplies; while, if he is deluding us, we ought
    certainly not to encourage him by sending them.
    Livy, xxiii. 13, § 5.

(34) Laws must punish, and punishment hurts.
    All laws therefore are hurtful.

(35) The sun is an insensible thing.
    The Persians worship the sun.
  ∴ The Persians worship an insensible thing.

(36) Some ores are not metals ; for they are not fluids,
    and some metals are not fluids.

(37) All the Grecian soldiers put the Persians to flight.
  ∴ Every Grecian soldier could rout the Persians.

(38) The resurrection of Jesus Christ is either an isolated

fact or else admits of parallel. But if it be an isolated fact, it cannot be rendered probable to one who denies the authority of Christianity; and, if it admit of parallel, it no longer proves what is required. Therefore it is either incapable of being substantiated or else makes nothing for the truth of Christianity.

(39) The resurrection of Christ in the flesh and His ascension into heaven were events either intrinsically incredible in their nature or not. If the former, the prevalent belief in them can only be accounted for by miracles; if the latter, they ought to be believed even without miracles. St. Aug., *De Civ. Dei*, xxii. 8.

(40) Only contented people are wise. Therefore the tramp contented in his rags is necessarily a wise man.

(41) Four-legged things are brutes.
       Tables are four-legged things.
   ∴ Tables are brutes.

(42) The apparent volcanoes in the moon are not volcanoes; for eruptions are produced by gases only, and there are no gases in the moon.

(43) To read the Scriptures is our duty. Therefore the Captain was wrong in punishing the helmsman for reading the Bible at the time when the ship struck.

(44) The divine law orders that kings should be honoured.
       Louis Quatorze is a king.
   ∴ The divine law orders that Louis Quatorze should be honoured.

(45) Those who desire the same object are unanimous.
       Caesar and Pompey both desire the same object, namely, supreme power.
   ∴ They are unanimous.

(46) Either the ministers left at home will be ciphers or they will not be ciphers. If they are ciphers, cabinet government, which is equivalent to constitutional government, will receive a rude blow. If they are not ciphers, the cabinet will be considering matters of the utmost importance in the absence, and the gratuitous absence, of two of its most important members. *The Standard*, Wed., June 5, 1878.

(47) One patent stove saves half the ordinary amount of fuel. Therefore two would save it all.

(48) One number must win in the lottery.
My ticket is one number.
∴ It must win.

(49) All good shepherds are prepared to lay down their lives for the sheep.
Few in this age are so prepared.
∴ Few in this age are good shepherds.

(50) You cannot define the sun: for a definition must be clearer than the thing defined, and nothing can be clearer than the source of all light.

(51) To give the monopoly of the home market to the produce of domestic industry ... must in almost all cases be either a useless or a hurtful regulation. If the produce of domestic can be brought there as cheap as that of foreign industry, the regulation is evidently useless ; if it cannot, it is generally hurtful. Adam Smith, *Wealth of Nations*, Bk. iv, ch. 2.

(52) Verberare est actio.
Ergo et vapulare.

(53) The ages of all the members of this family are over 150.
The baby is a member of this family.
∴ Its age is over 150.

(54)   Romulus must be an historical person; because it is not at all likely that the Romans, whose memory was only burdened with seven kings, should have forgotten the most famous of them, namely, the first.

(55)   Deus aut vult tollere mala et non potest, aut potest et non vult, aut neque vult neque potest, aut et vult et potest.   Si vult et non potest, imbecillis est, quod in Deum non cadit; si potest et non vult, invidus, quod aeque alienum a Deo; si neque vult neque potest, et invidus et imbecillis est, ideoque neque Deus; si et vult et potest, quod solum Deo convenit, unde ergo sunt mala? aut cur illa non tollit?   Epicurus apud Lact. *De Ira Dei*, 13.

(56)   Gold is yellow.   But gold is also heavy; and heavy is not-yellow.

∴ Gold is yellow and not-yellow.

(57)   Professor Joseph Jastrow, of the University of Wisconsin, proposes the following little problem in logic as a means of testing diversity of opinion:

Granted that A is B, to prove that B is A.

B (like everything else) is either A or not A.

If B is not A, then by our first premiss we have the syllogism—

A is B,

B is not A,

∴ A is not A, which is absurd.

Therefore B is A.

Is this reasoning correct, he asks, or is it not?

The Academy, Jan. 16, 1897.

(58)   Perhaps I may be allowed in return to propound a sophism for the consideration of the Professor. In the unlikely event of his failing to solve it, he must be held responsible for the consequences to public morality.

An indifferent act is not-right.
An indifferent act is not-wrong.
∴ Not-wrong is not-right.
∴ (By contraposition) Right is wrong.

The Academy, Jan. 23, 1897.

(59)   Quando nos sumus, mors non est ;
quando mors est, nos non sumus.
Mors ergo nihil ad nos.

Epicurus apud Lact. *Div. Inst.* III. 17.

(60)   Nam si homicida nefarius est, quia hominis ex-
stinctor est, eidem sceleri obstrictus est, qui se
necat, quia hominem necat.   Lact. *Div. Inst.*
III. 18.

(61)   Consulat unusquisque affectus suos : jam intelleget,
neminem posse sine ira et castigatione imperio
subjugari.   Ubi ergo ira non fuerit, imperium
quoque non erit.   Deus autem habet imperium.
Ergo et iram, qua constat imperium, habeat
necesse est.   Lactantius, *De Ira Dei,* 23.

(62)   Doctors say it is healthy to live on gravel; but
this is impossible, for gravel is very indigestible.

(63)   How do you know when a verb ought to be put
into the subjunctive in the oblique oration ?
When it is in a dependent clause.
How do you know that a clause is dependent ?
When the verb is in the subjunctive.

(64)   Whoever divulges the mysteries to the uninitiated
commits impiety.
The hierophant divulges the mysteries to the
uninitiated.
∴ The hierophant commits impiety.

D L. VII, § 186.

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